Space Missions and Systems 2020: Homework #1

A spacecraft is in an eccentric, equatorial orbit about the Earth. A tracking station, located on the Earth equator, is collecting range and range-rate observables. The *a priori estimate* of the probe state vector and the associated uncertainty, in the inertial, geocentric frame at the initial reference time $t_0 = 0$, are:

	Value	Uncertainty
x	$-10843\ km$	$20\ km$
y	$380\ km$	$20\ km$
v_x	$-0.2412\ km/s$	$1\ m/s$
v_y	$-6.9066\ km/s$	$1\ m/s$

The spacecraft can be approximated as a flat plate with an area of $10\ m^2$ and a mass of $100\ kg$. The plate is always nadir-pointed (the normal $\vec{n}$ of the plate points towards the center of the Earth).

The spacecraft is subject to the gravity of the Earth, assumed as a spherically symmetric body ($GM_\oplus = 398604 \pm 10\ km^3/s^2$), and to a non-gravitational acceleration caused by the solar radiation pressure. Assume that all the radiation is absorbed (this will certainly heat up the plate, but let us suppose that the IR radiation is emitted symmetrically on both sides) and that the Sun lies on the negative x-axis (neglect the effects of the finite Earth-Sun distance).

he position of the tracking station is assumed to be perfectly known. At the reference time the angle between the station's position vector and the X axis of the reference frame is $\lambda_0 = 40°$. See next figure for the geometry of the problem.

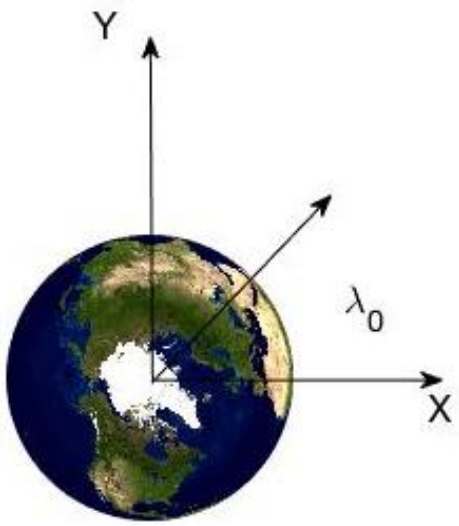

Figure 2 - North pole view of the Earth. λ_0 not in scale.

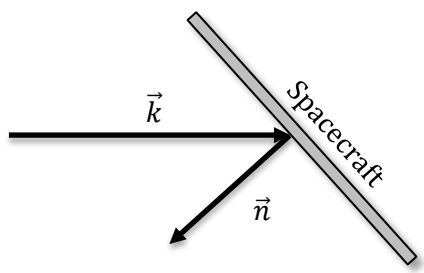

Figure 1 - Spacecraft geometry. $\vec{n}$ is the spacecraft surface normal, $\vec{k}$ is the Sun-Spacecraft unit vector.

(1) Use the range observables (accuracy $< 100\,m$) and range-rate observables (accuracy
$< 1\,cm/s$) provided by the ground station to obtain the minimum variance estimate of the probe's state vector, the solar flux and the Earth's GM at the reference epoch. Comment the results.

(2) Compute and plot the behavior of the argument of pericenter and eccentricity on a period of 1 year.

(3) Compute and plot the evolution of the uncertainty over 1 month of all the estimated parameters. Comment the results.

(4) What kind of measurement (range or range-rate) is the most useful for the estimation of Φ and $GM_{\oplus}$? Justify your answer.

Additional information:

- The observables are provided in the files "range.txt" and "rangerate.txt": the first column contains the observation time (in seconds after t_0), the second column is the corresponding observed value (in km for the range and km/s for the range-rate).
- Earth sidereal rotation period (counterclockwise): 23 hours, 56 min, 4 sec.
- Earth equatorial radius: 6371 km.
- In the computation of the solar radiation pressure consider the spacecraft as purely absorptive and always in light (no shadowing/eclipse effects). The solar radiation pressure in this case is given by:

$$\vec{F}_{srp} = \frac{\Phi}{c} A_p \, \vec{k}$$

Where:

- Φ : Solar flux
- c : speed of light $299792.458\,km/s$
- A_p: Projected area along $\vec{k}$ (note: this is always non-negative)
- $\vec{k}$: Sun-spacecraft direction ($\vec{k}$ is a unit vector)

Upload a working computer code (Matlab or any other programming language) and a short note (a pdf file) with the mathematical procedure, results, comments, and figures on the Google Classroom of the course. The deadline is **5 April 23:59 CEST.**

Be concise and go straight to the point.

In the note indicate your **first name**, **last name**, and **student id** (a.k.a., "numero di matricola").

<u>Tips on Matlab:</u>

- For better performances consider using ode113 instead of ode45
- For integration purposes use at least RelTol=XE-10, AbsTol=XE-13;
- ALWAYS put labels and units on the plot axes!