Interplanetary Navigation

Examples from the Cassini/Huygens Mission

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Agenda

- Overview
  - Objectives
  - Functions
- Trajectory Design
  - Direct versus Indirect Trajectories
  - Flybys versus orbiters
- Estimation
  - General Techniques
  - Measurements
  - Models
  - Typical Results
- Control
  - General Techniques
  - Propulsion/Control Systems
  - Navigation Simulation
  - Typical Results

Overview

- Navigation Objective: Delivery instrument fields of view to the desired location subject to the constraints of the spacecraft and ground system design.
    - Support the design of the reference trajectory
        - The reference trajectory provides a “road map” for the execution of the mission
    - Estimate the “state” at all times during the mission
        - “State” may include:
            - spacecraft state,
            - satellite ephemerides,
            - satellite masses,
            - model parameters
    - Control the mission trajectory

Task Phases

- Planning and Prediction
  - Planning the trajectory
  - Defining the navigation strategy
  - Prediction of the capabilities (Accuracy, propellant,…)
- Execution
  - Collecting the measurements
  - Processing the measurements and estimating the model parameters
  - Correcting the trajectory
- Reconstruction
  - After the fact estimating where the spacecraft has been and the associated model parameters

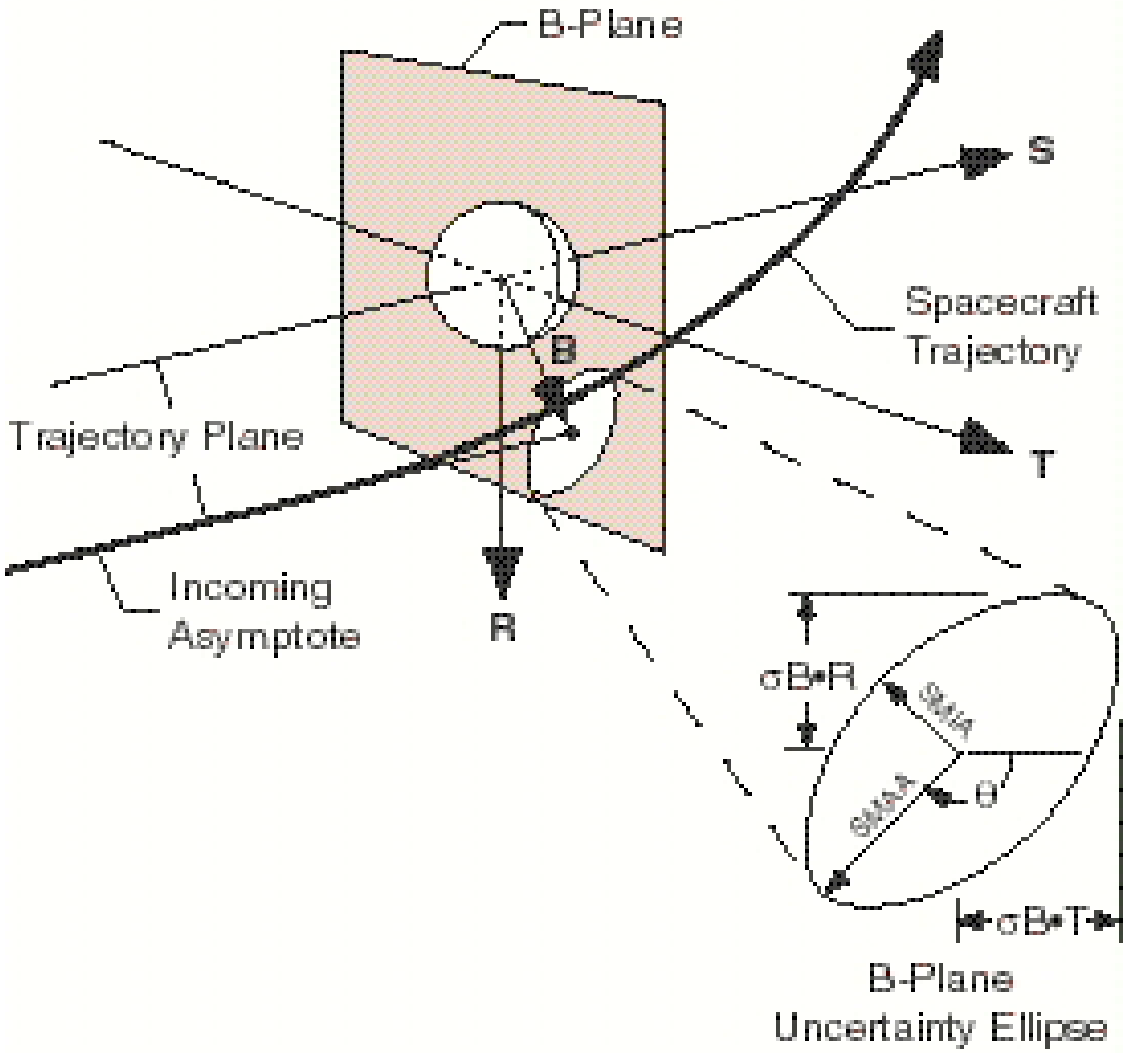
Trajectory Design

- Objective: Find a trajectory which meets the scientific objectives within the capability of the launch vehicle and the spacecraft.
    - Considerations:
        - Launch Window - The duration of the time interval that launch can occur
            - Can vary from instaneous to hours
                - Cassini's daily launch window varied from 5 minutes to 140 minutes
        - Launch Period - The number of days during which a launch can occur
            - Cassini's launch period for the primary mission was about 1 month.
        - Launch Opportunities - The number of available launch periods
            - Cassini identified 3 separate launch periods. Significant variations in the fundamental trajectory for different launch opportunities

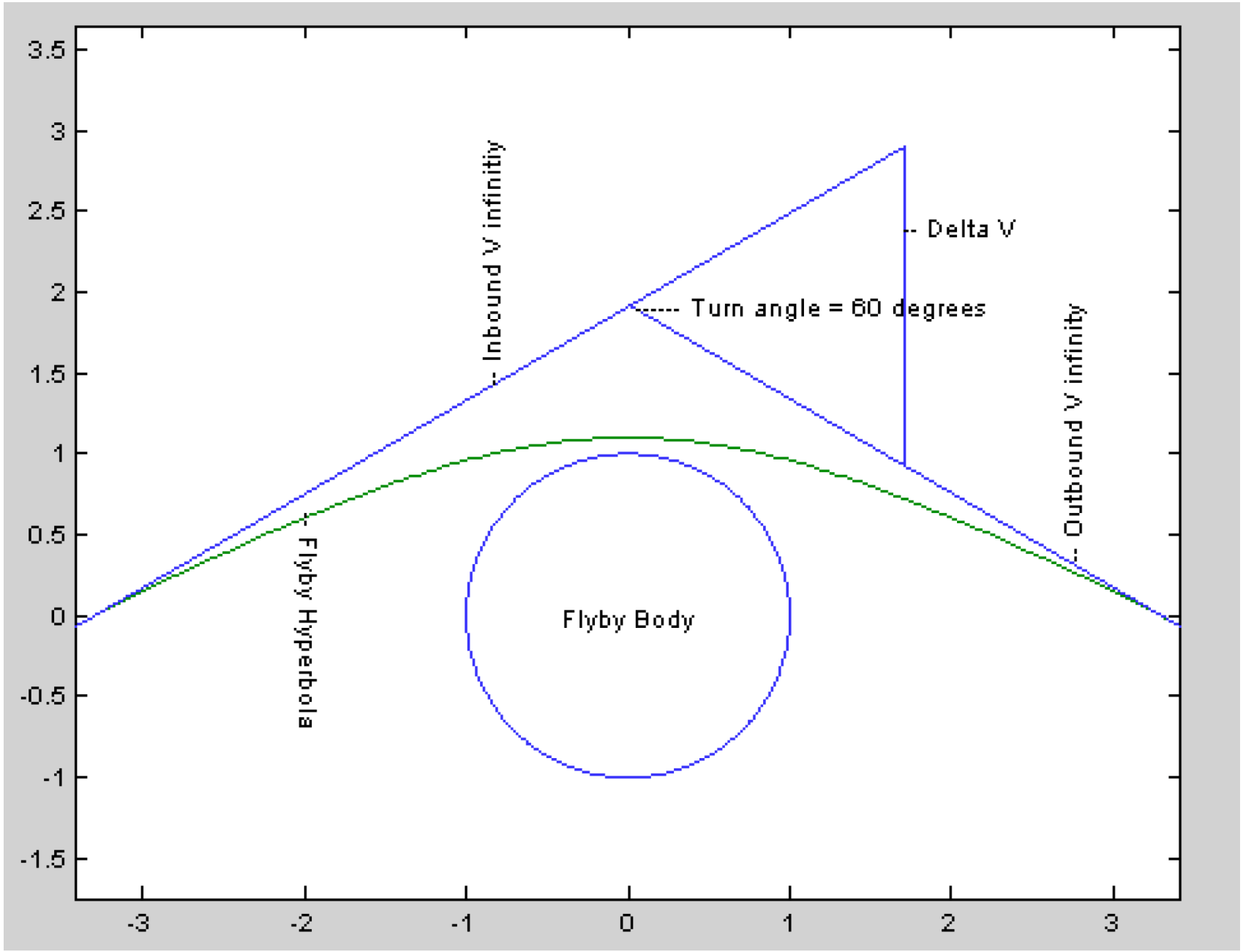
Trajectory Design (Continued)

- Direct versus Gravity Assist Trajectories
  - Direct trajectories proceed from launch to their target with only propulsive maneuvers to modify the flight path
    - Minimizes the transfer time (cost) from launch to the target.
  - Gravity assist trajectories use the “sling shot” effect to change (increase or decrease) the orbital energy and inclination.
    - Maximizes the payload capability at the expense of time.
      - Cassini used two Venus flybys, one Earth flyby and one Saturn flyby to increase the orbital energy by 21 km^2/sec^2, but took 6.7 years to reach Saturn
      - Voyagers 1 & 2 used a direct trajectory that took about 4.1 to reach Jupiter
- Low Thrust Trajectories
  - Use low thrust - high impulse to decrease transfer time and/or increase payload

B-Plane


B-Plane
S
Spacecraft
Trajectory
B
Trajectory Plane
T
Incoming
Asymptote
R
σB•R
σB•T
θ
B-Plane
Uncertainty Ellipse

Flyby Geometry



Gravity Assist

Changing an orbit parameters without propellant

V_p = "Planet" _ Velocity

V^- = Spacecraft _ velocity _ before _ the _ flyby

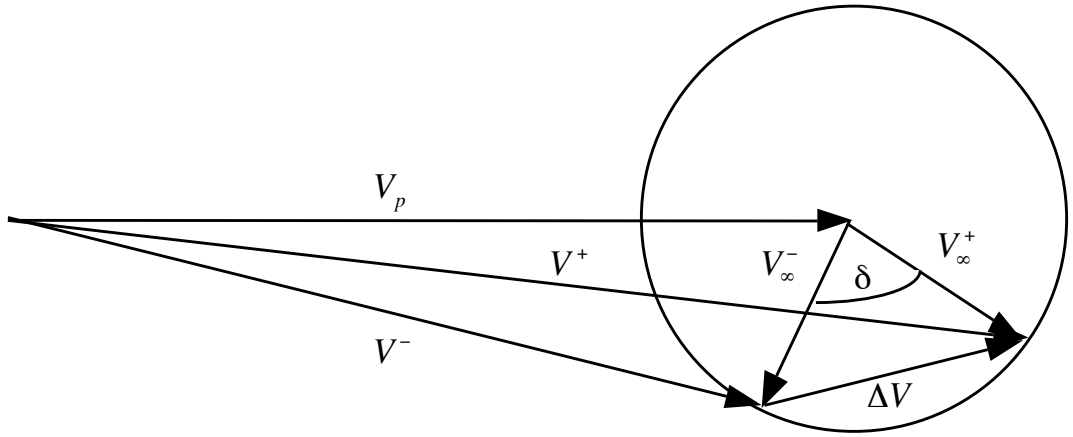
V_∞^- = Asymptotic _ velocity _ before _ the _ flyby

V_∞^+ = Asymptotic _ velocity _ after _ the _ flyby

δ = Turn _ angle _ during _ the _ flyby

ΔV = velocity _ change

V^+ = Spacecraft _ velocity _ after _ the _ flyby



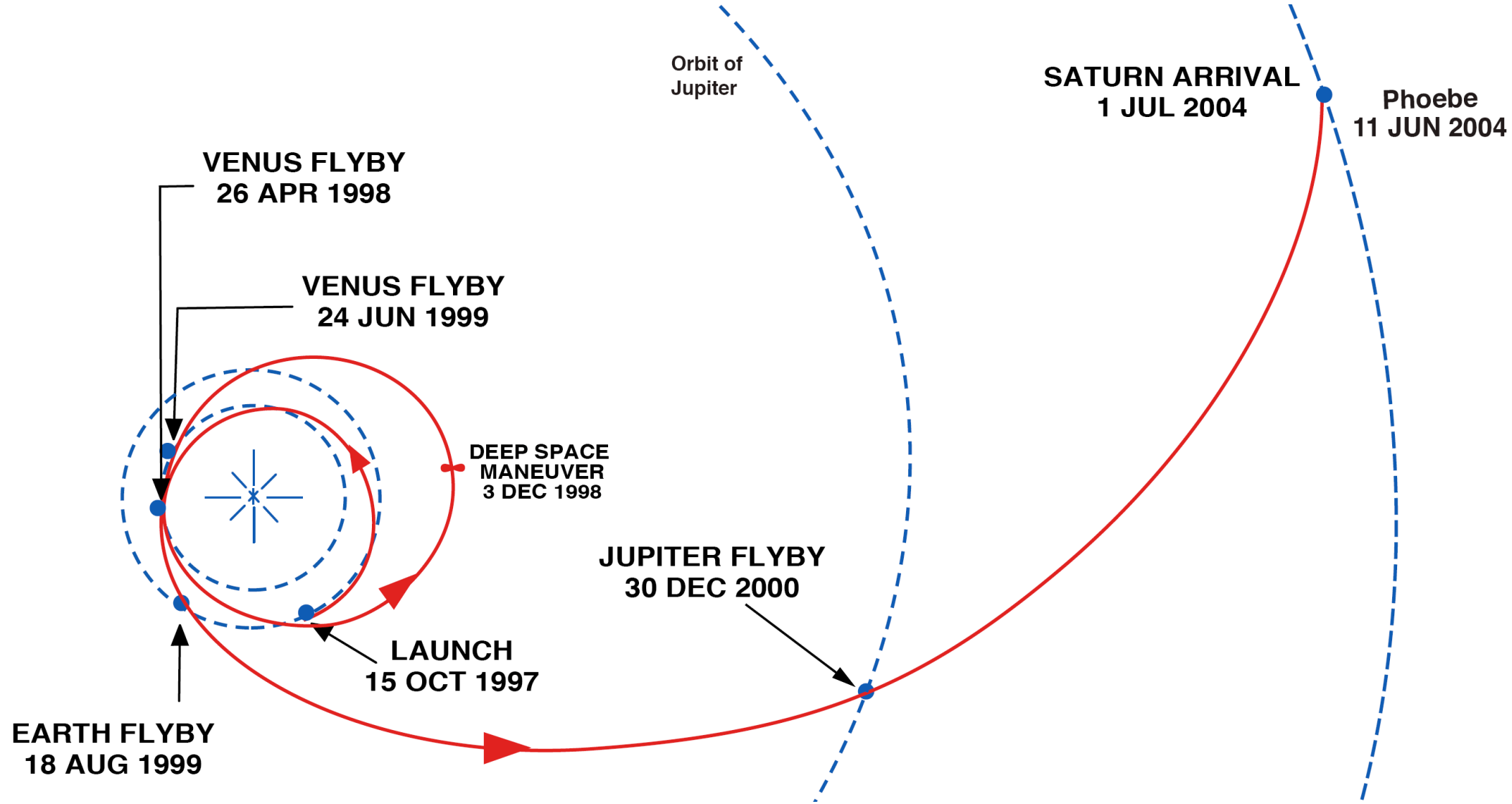
$$\Delta V = 2V_\infty Sin(\delta/2)$$

$$Sin\left(\delta/2\right) = 1/e$$

$$e = 1 + \frac{r_p V_\infty^2}{\mu}$$

Question: Where did the energy change come from ?

Cassini Interplanetary Trajectory



Cassini Trajectory

• Both the interplanetary trajectory and the tour make use of gravity assists to achieve the objectives with a minimum of propellant.

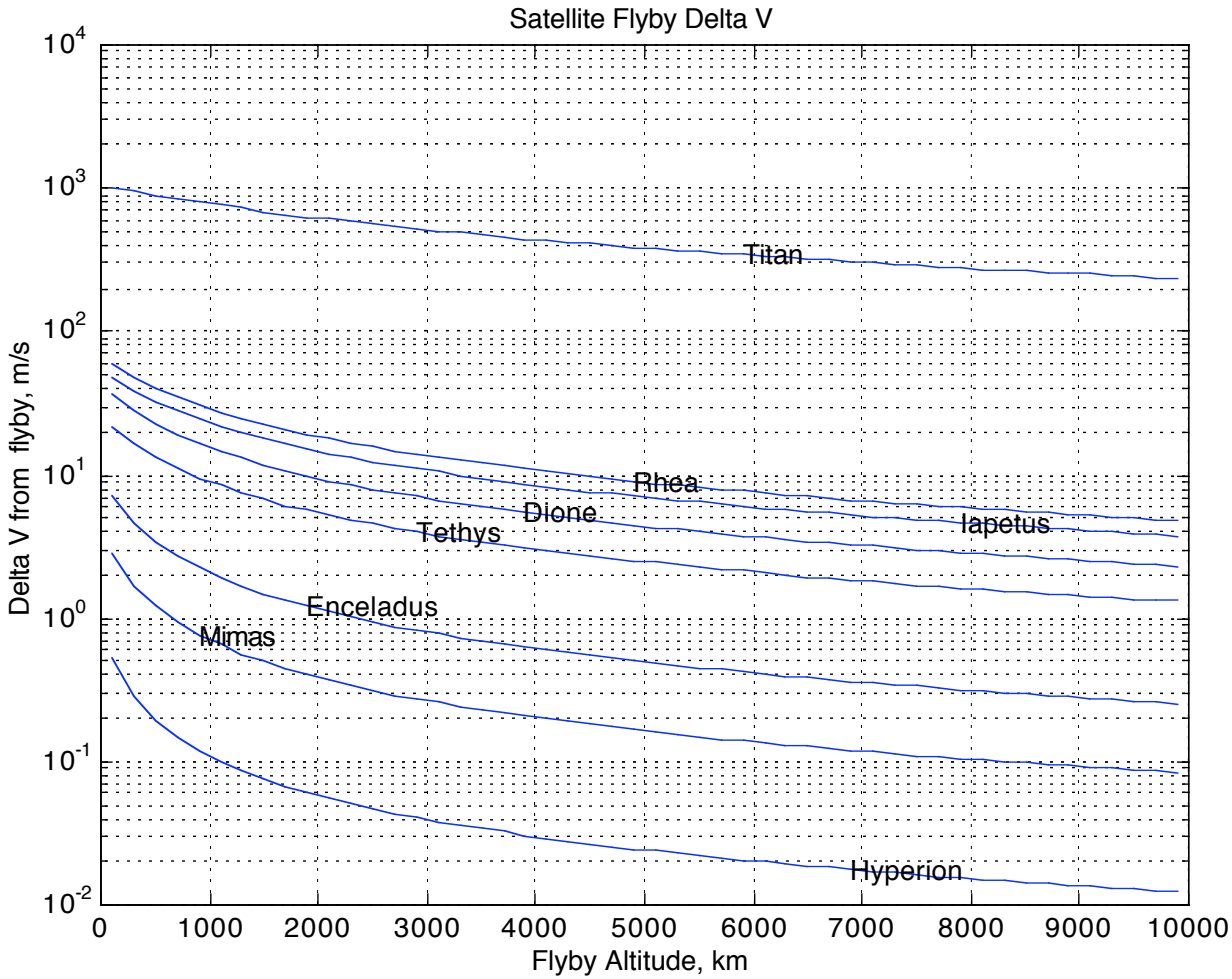
Interplanetary Flyby Summary

Body	date	Flyby Alitude, km	Velocity at Periapsis, km/sec	Turn Angle. Deg	Gravity Assist, km/sec
Launch	10/15/97 0:00				
Venus	4/26/98 13:44	338	11.75	71.33	7.03
Venus	6/24/99 18:24	624	13.63	41.56	6.66
Earth	8/18/99 3:03	1,154	19.03	19.71	5.48
Jupiter	12/30/2000 10.35	9,721,940	11.6	12.21	2.22
				Total	21.39

• At launch the spacecraft capability was about 2 km/sec

Tour Flybys

• Titan is the only satellite of Saturn with sufficient mass to significantly change Cassini's trajectory



V_{inf} = 5.5 m/sec

Total ΔV from Titan during the tour is about 32 km/sec

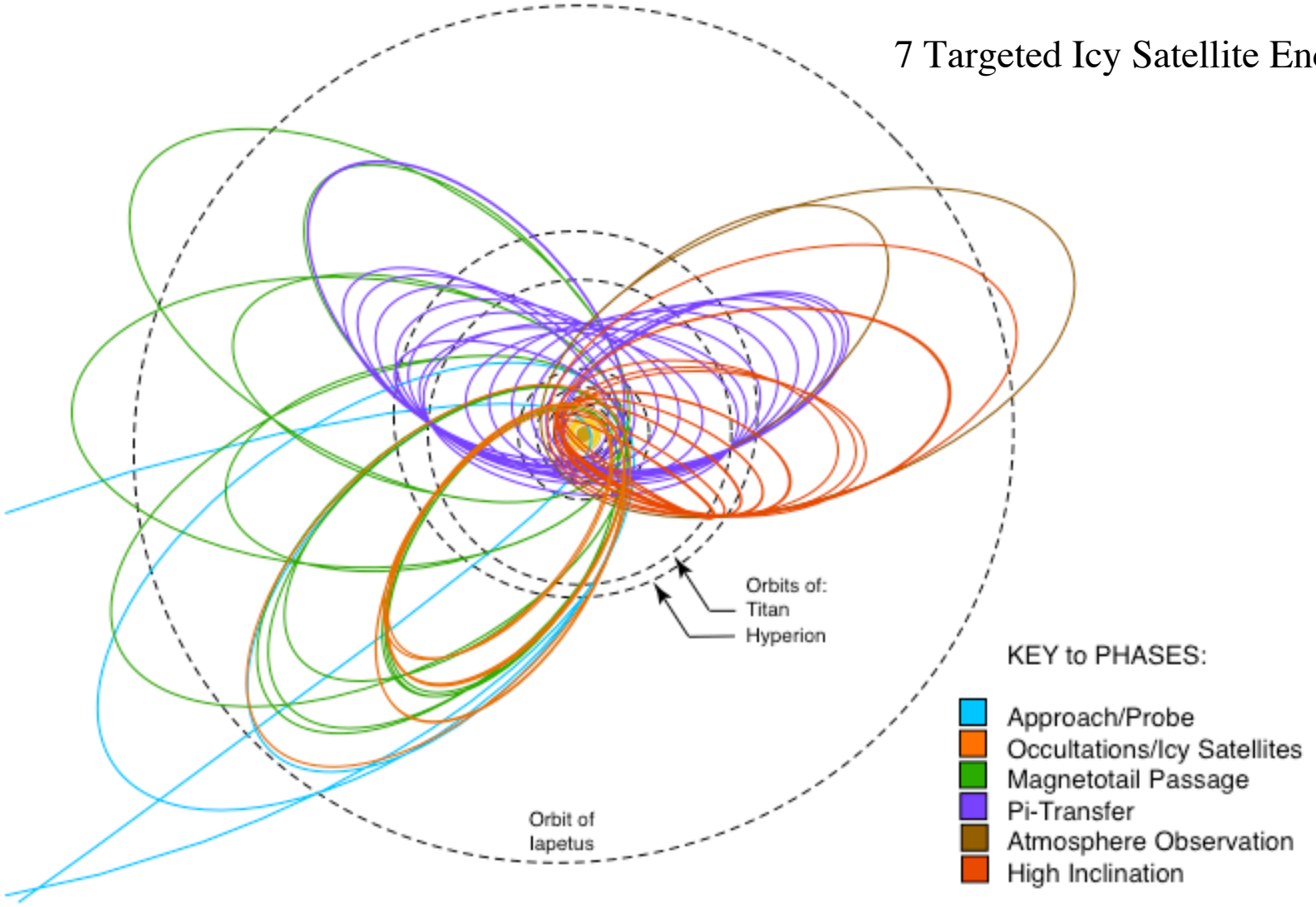
CASSINI MISSION SATELLITE TOUR

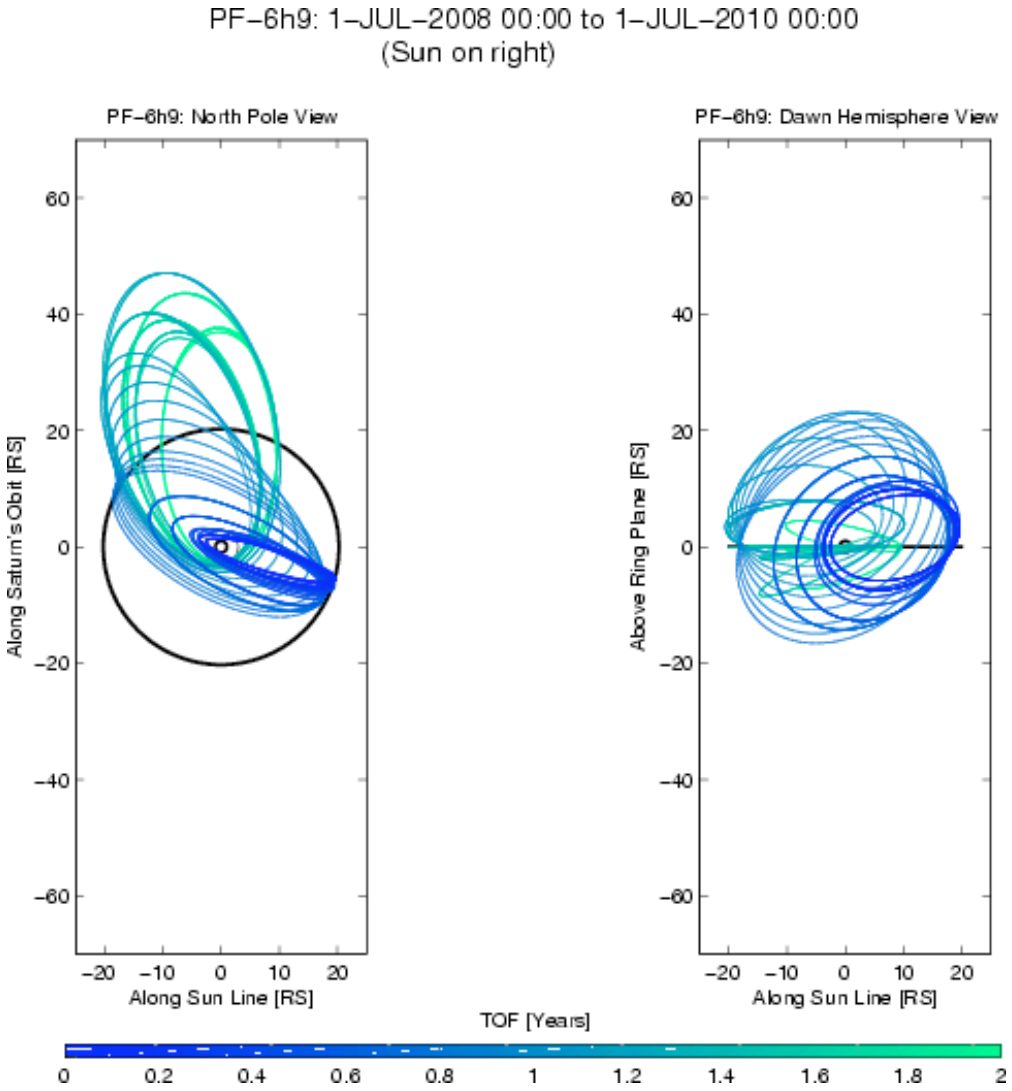
Tour - All Orbits

74 orbits

44 Titan encounters

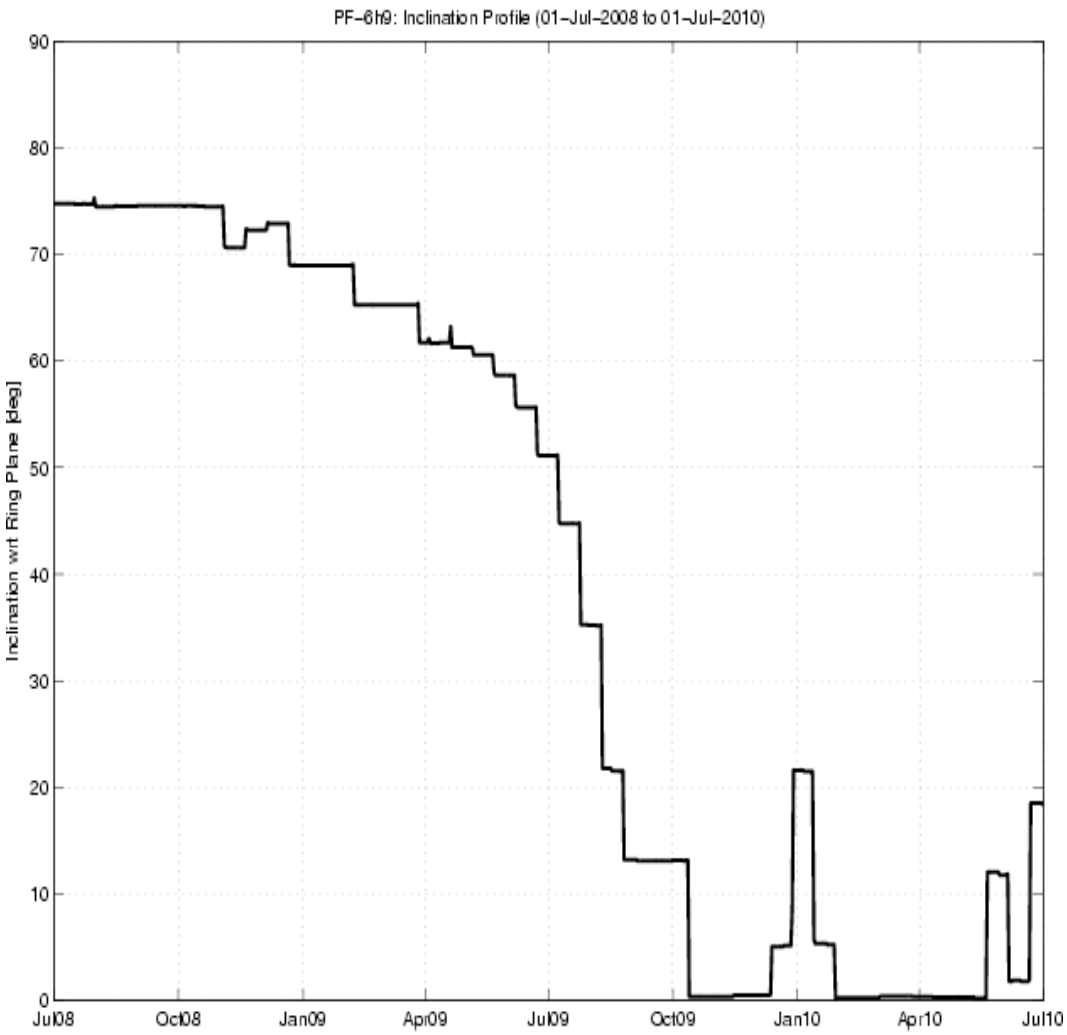
7 Targeted Icy Satellite Encounters





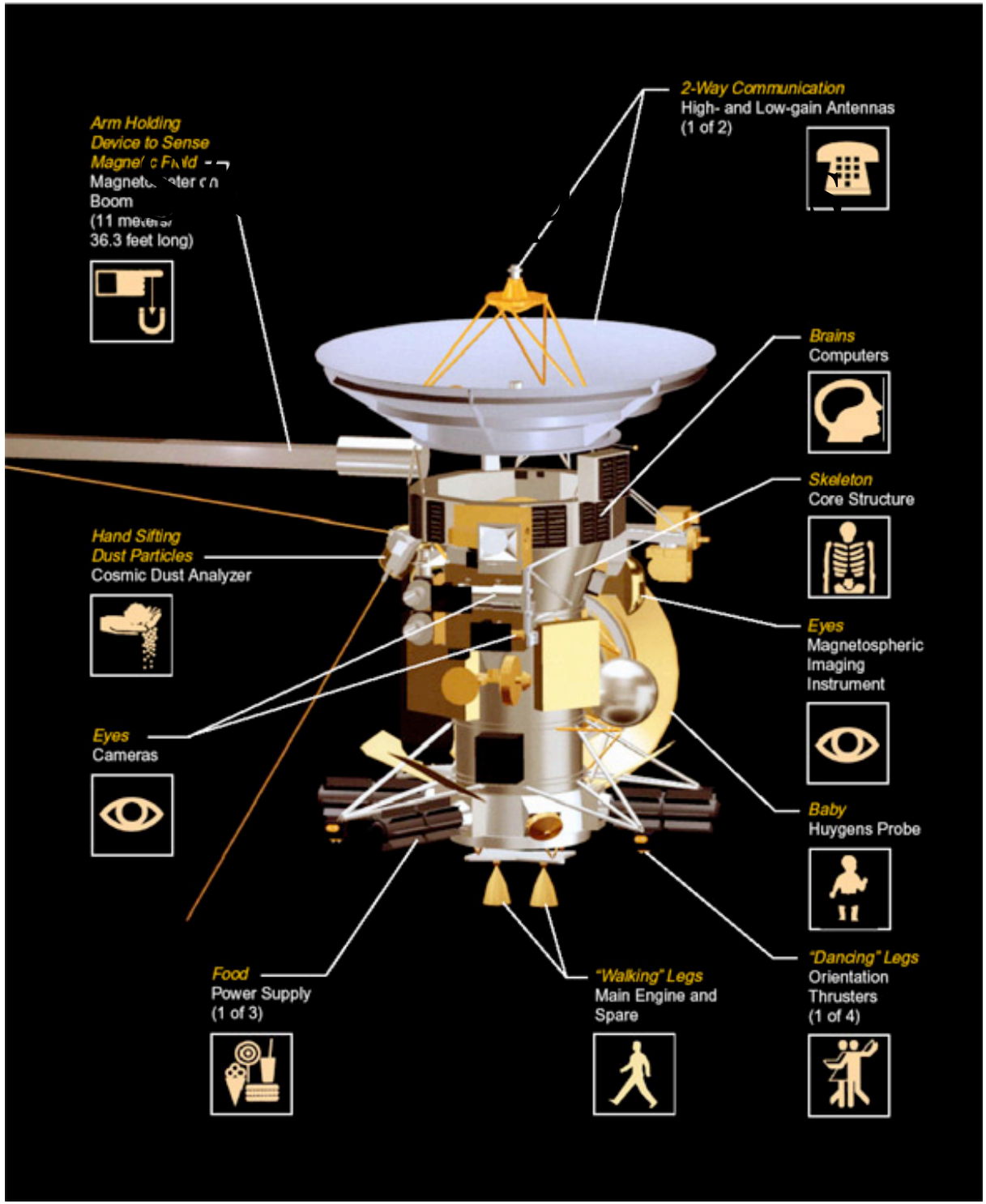
Extended Mission

- 2 years
- 26 Titan encounters
- 7 Enceladus encounters



Cassini Spacecraft
and other images

Cassini Assembly at JPL


2-Way Communication
High- and Low-gain Antennas
(1 of 2)
Arm Holding
Device to Sense
Magnetic Field
Magnetometer on
Boom
(11 meters
36.3 feet long)
Brains
Computers
Skeleton
Core Structure
Hand Sifting
Dust Particles
Cosmic Dust Analyzer
Eyes
Magnetospheric
Imaging
Instrument
Eyes
Cameras
Baby
Huygens Probe
"Dancing" Legs
Orientation
Thrusters
(1 of 4)
Food
Power Supply
(1 of 3)
"Walking" Legs
Main Engine and
Spare

On Launch pad

Huygens Probe

Navigation Inputs to Mission Design

- Measurement Requirements
    - What types of measurements are required
        - Radio, Optical, Radar
    - Measurement accuracy
    - Measurement frequency
- Maneuver Requirements
    - Maneuver frequency
    - Maneuver accuracy
    - Propulsion system requirement
- The results can generate requirements on:
    - Spacecraft
    - Ground system
    - Navigation software
    - Mission sequences

Cassini Navigation Inputs to Mission Design

- Spacecraft
  - Transponder requirements
    - Frequency (X-Band, Ka-Band,VLBI tones)
  - Camera requirements
    - Reliability
  - Attitude control
    - Coupled versus Uncoupled attitude control thrusters (lost this battle)
  - Propulsion system
    - Number of repressurizations (long story)
    - Maneuver durations (min/max)
  - Data storage
    - Number of optical navigation Images
- Ground System
  - Number of tracking passes (function of mission phase)
  - Ranging accuracy requirement
  - Optical Navigation image processing

Cassini Navigation Inputs to Mission Design (Continued)

- Navigation Software
  - Launch Polynomials
  - Small forces data interface
  - Automated maneuver processing
  - Solar pressure model
  - Turn ΔV prediction
  - Chained maneuver computation
  - Satellite ephemeris numerical accuracy

Estimation

- For Interplanetary Navigation this is usually referred to as Orbit Determination

Estimation

- The foundation of navigation is having a mathematical models of the physics controlling the trajectory of the spacecraft and the measurements.
- Estimation is the process of determining the “best estimates” of the parameters of the models.
    - Examples of the parameters include
        - State (position and velocity) of the spacecraft at a specified time.
        - State and mass of the planets and satellites
        - Magnitude and direction of a non-gravitational force acting on the spacecraft
        - Accuracy of a measurement type

Mathematics

- For spacecraft orbit estimation a linear least square approach is used.
- Many different formulations but they all have certain elements in common
    - Linearization - problems are non-linear and require linearization to determine a closed form solution
        - Iteration required to reach “final” solution
    - Minimization of penalty function, usually quadratic
    - Use of Gaussian statistics

Question: Why are Gaussian statistics used ?

Simple Formulation

- Estimate the n - component state vector, x

- Using p - component measurements, z

  Where the measurements have random errors, υ (independent of x)

- The measurements and the state are related by

$$z = Hx + \upsilon$$

  Where H is a known (pxn) transition matrix

- Assume that the measurement errors, υ, have a zero mean, $E\{\upsilon\} = 0$,

  and have variance $E\{\upsilon\upsilon^Y\} = R$ (a known (pxp) positive matrix)

- The initial estimate of the state before the measurements, $\bar{x}$

  with variance $E\{(x - \bar{x})(x - \bar{x})^T\} = M$ (a known (nxn) positive matrix)

Applied Optimal Control

Bryson, A. E. Jr., Ho Y-C

Blaisdell Publishing Company

Simple Formulation (Continued)

Problem: Find the value, $\hat{x}$, that minimizes

$$J = \frac{1}{2}\left[\left(x - \bar{x}\right)^T M^{-1}(x - \bar{x}) + \left(z - Hx\right)^T R^{-1}\left(z - Hx\right)\right]$$

the differential is:

$$dJ = dx^T [M^{-1}(x - \bar{x}) - H^T R^{-1}(z - Hx)]$$

for $dJ = 0$, for arbitary dx^T, requires that the coefficient of dx^T must vanish,

$$\hat{x} = \bar{x} + PH^T R^{-1}(z - H\bar{x})$$

where

$$P^{-1} = M^{-1} + H^T R^{-1} H \quad \text{(nxn inverse)}$$

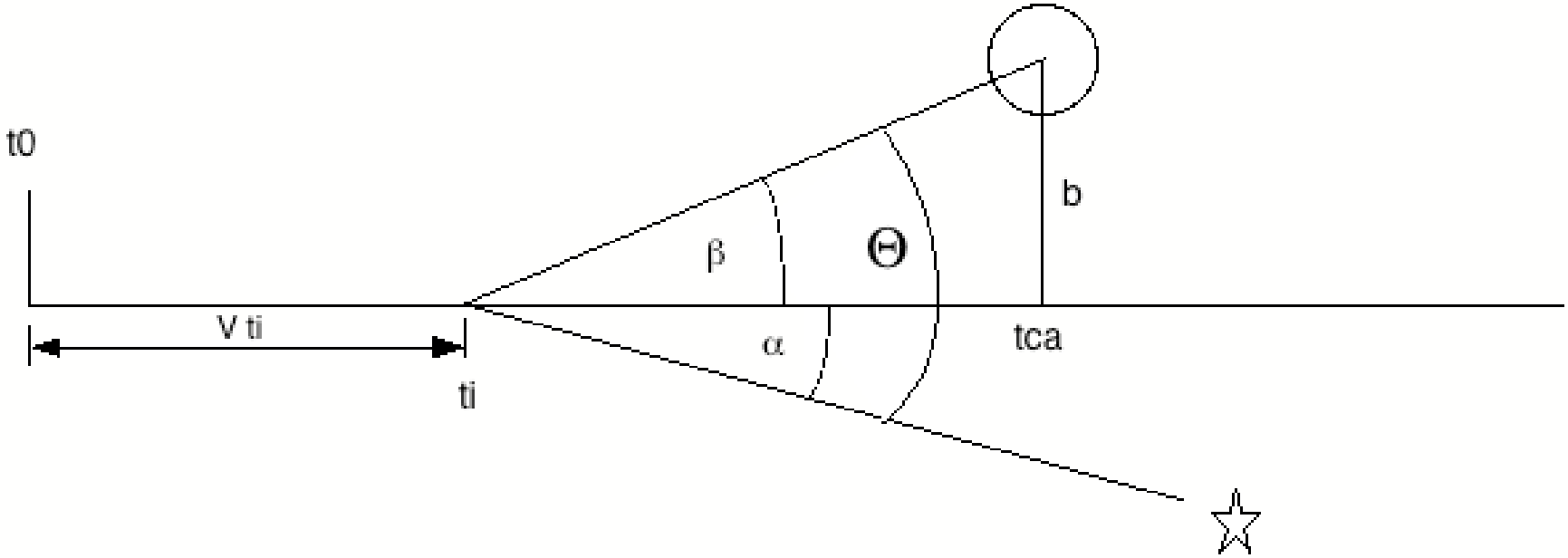
Note: for convience the follows relationships are convient if p < n:

$$P = M - MH^T \left(HMH^T + R\right)^{-1} HM \quad \text{(pxp inverse)}$$

$$PH^T R^{-1} = MH^T \left(HMH^T + R\right)^{-1} \quad \text{(pxp inverse)}$$

Simple Example

Estimate the flyby conditions for a asteroid flyby using optical navigation



Estimate the state vector $[b \quad v \quad t_{ca} \quad \alpha]$

Using images at times, ti, to estimate the angle between a known star and the center of the asteroid

Formulation

The measurement, Θ at time t_i, is related to the state by :

$$\tan(\Theta) = \tan(\alpha + \beta) = \frac{b}{V(t_{ca} - t_i)}$$

therefore

$$\Theta = -\alpha + \tan^{-1}\left(\frac{b}{V(t_{ca} - t_i)}\right)$$

While the motion is linear, the measurement is non - linear,

therefore linearize about an initial state estimate $\left[\bar{b}, \bar{V}, \bar{t}_{ca}, \bar{\alpha}\right]$

$$\mathrm{dz_i} = \Theta_i - \bar{\Theta}_i = \left[H_{1i}, H_{2i}, H_{3i}, H_{4i}\right]dx + \upsilon_i$$

The measurement partials at each measurement time, t_i are :

$$H(1,i) = \frac{\partial\Theta}{\partial b} = \frac{\bar{V}(\bar{t}_{ca} - t_i)}{\bar{r}^2}\ ,\ H(2,i) = \frac{\partial\Theta}{\partial V} = -\frac{\bar{b}(\bar{t}_{ca} - t_i)}{\bar{r}^2}\ ,\ H(3,i) = \frac{\partial\Theta}{\partial t_{ca}} = -\frac{\bar{b}\,\bar{V}}{\bar{r}^2}\ ,\ H(4,1) = \frac{\partial\Theta}{\partial\alpha} = 1$$

where $\bar{r}^2 = \bar{b}^2 + \bar{V}^2(\bar{t}_{ca} - t_i)^2$

Formulation (continued)

Let :

$$dz = \begin{bmatrix} \Theta_1 - \overline{\Theta}_1 \\ . \\ . \\ \Theta_{n1} - \overline{\Theta}_{n1} \end{bmatrix} \quad H = \begin{bmatrix} H_{1i}, H_{2i}, H_{3i}, H_{4i} \\ . \\ . \\ H_{1n}, H_{2n}, H_{3n}, H_{4n} \end{bmatrix} \quad dx = \begin{bmatrix} b - \overline{b} \\ V - \overline{V} \\ t_{ca} - \bar{t}_{ca} \\ \alpha - \overline{\alpha} \end{bmatrix} \quad \upsilon = \begin{bmatrix} \upsilon_{\mathrm{i}} \\ . \\ . \\ \upsilon_n \end{bmatrix}$$

Then we have

$$dz = Hdx + v$$

the measurement uncertainty is :

$$R = \begin{bmatrix} r_1, ...0 \\ . \\ . \\ 0, ...r_n \end{bmatrix}$$

The uncertainty in the initial state is :

$$M = \begin{bmatrix} \sigma(dx_1)^2, \sigma\left(dx_1 dx_2\right), ., . \\ \sigma\left(dx_1 dx_2\right), \sigma(dx_2)^2, ., ; \\ . \\ ., ., \sigma\left(dx_3 dx_4\right), \sigma(dx_4)^2 \end{bmatrix}$$

Formulation (continued)

The best estimate of the initial state estimate is then given by:

$$d\hat{x} = d\bar{x} + PH^T R^{-1}\left(dz - Hd\bar{x}\right)$$

where $P = \left(M^{-1} + H^T R^{-1} H\right)^{-1}$

The best estimate can be obtained in two ways

1. Batch processing - a set of measurements are processes in a single batch
2. Sequential Processing - Each measurement is processed individually.

Since the mesurement process is non-linear, the initial state estimate should be updated with the "optimal" corrections until the corrections reach some minimal value

Measurements

- For interplanetary missions, the following measurements are used

  - Doppler - frequency shift in the carrier - measures the range rate from the ground antenna to the spacecraft antenna

    - Cassini uses X-Band with a typical accuracy of 0.1 mm/sec (1σ)

    - Since the Earth's motion affects the range rate, information on the right ascension and declination can be obtained.

    - Accuracy depends up the stability of the ground reference signal (clock), spacecraft down link is coherently locked to the uplink signal.

    - Major error source is the interplanetary plasma

  - Range - Measurement of the round trip light time

    - Range typical accuracy of 1 meters (1σ)

    - Path length dependent

Measurements (Continued)

- Optical Images - measurement of the angle between a known star and the optical center of a target body
    - Star location estimated to about 0.25 pixels (1σ)
    - Accuracy of the estimation of dynamical center depends very much upon the body and the phase angle
        - Typical problems are:
            - Atmosphere - Titan
            - Irregular shape - Hyperion
- VLBA - Very Long Baseline Array
    - Measures the angle spacecraft and celestial radio sources
    - Accurate estimate of the angular location of the spacecraft in the radio frame. Typical accuracy of 0.1 nanoseconds in the delay time at two stations.
    - Primarily used of Saturn ephemeris improvement
- Angles - The antenna angles
    - Typically useful when very close to the station (Earth orbiters and post-launch)

Models

- Estimating the orbit of the spacecraft requires models describing
  - The forces acting on the spacecraft
  - The measurement

Forces acting on the spacecraft

Gravitation Forces

- Central body
- Third body
- Harmonics
- Relativity

Non-Gravitational Forces

- Thrusters
  - Orbit corrections
  - Attitude control
- Solar pressure
- Thermal radiation
- Drag

Models (Continued)

Measurement Models

Planet and satellite ephemerides

Range biases

Troposphere & Ionosphere

Earth orientation

Station locations

Relativity

Optical pointing

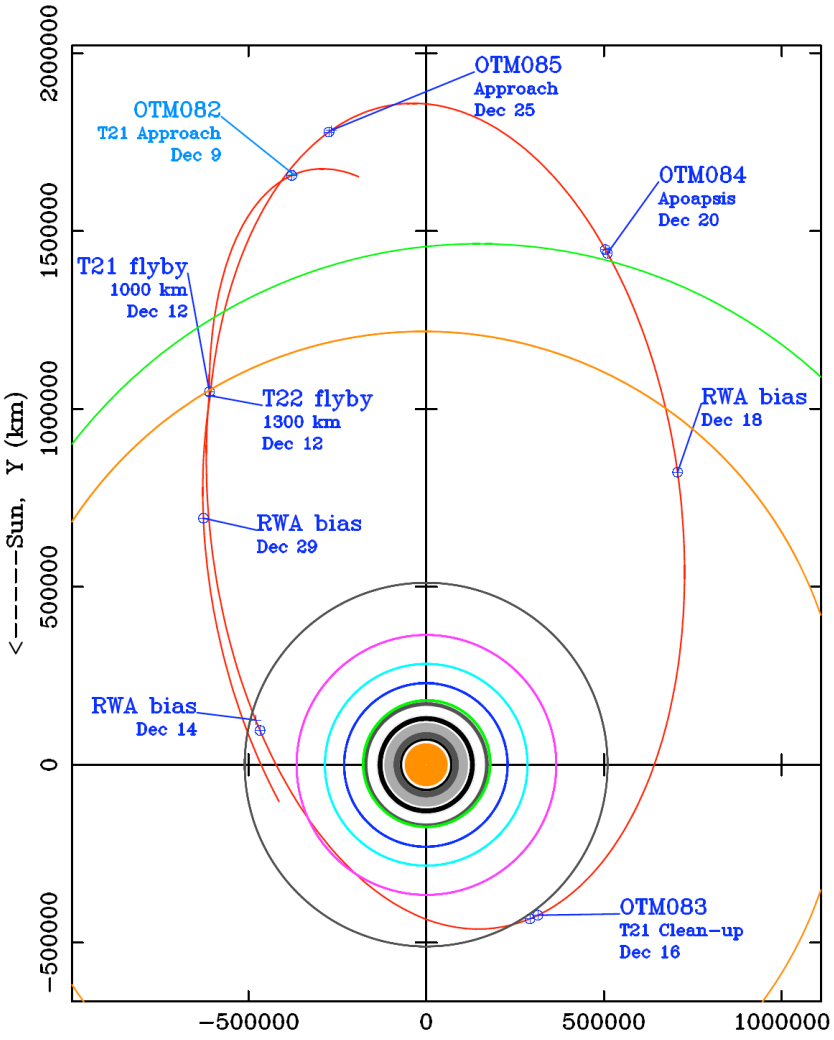
Star Catalog

Typical Cassini Orbit Determination Accuracy

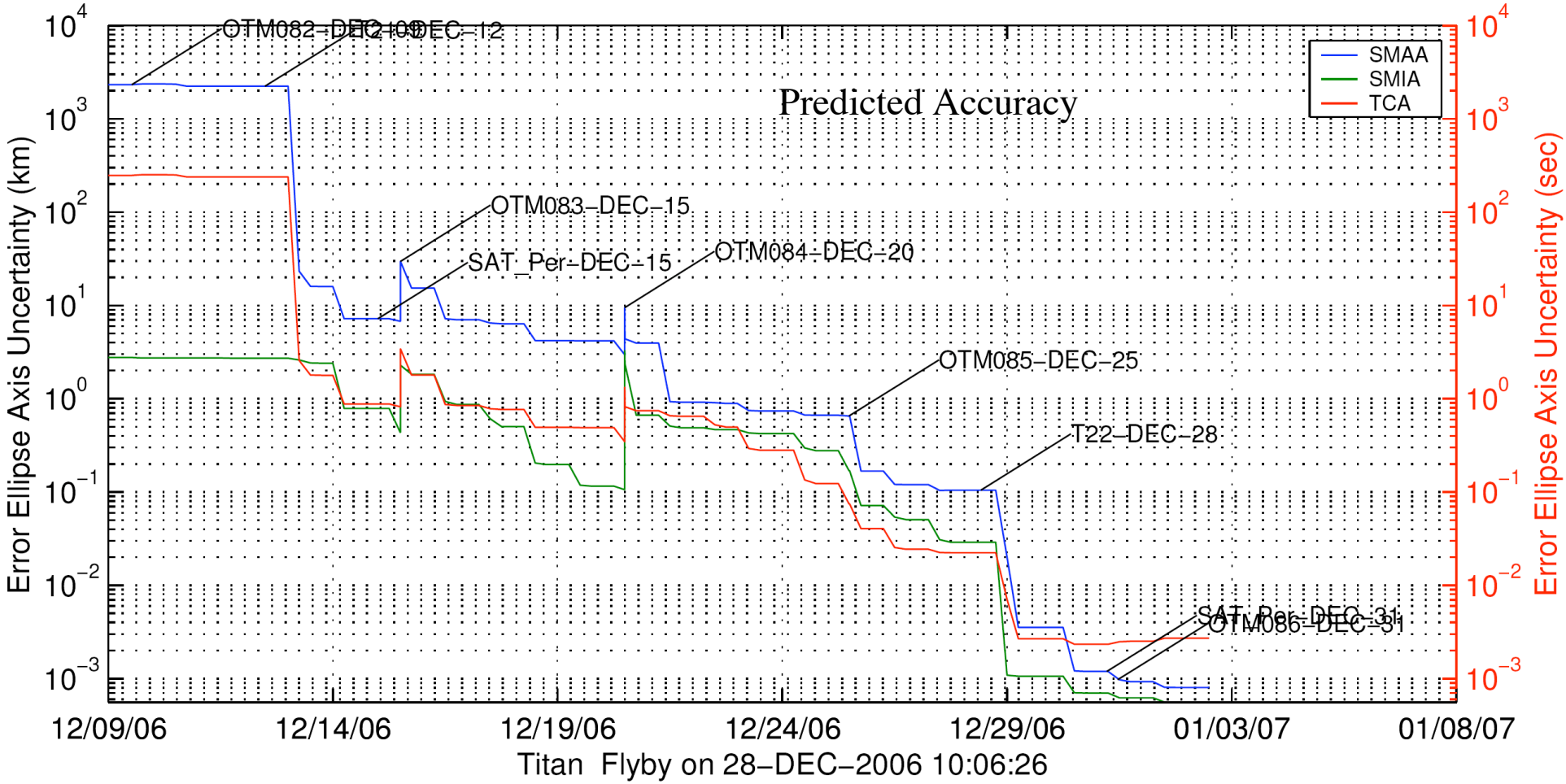
- 22th Titan Encounter (T21) to the 23st Titan encounter (T22)
- Data arc starts at the apoapsis before the T21 encounter and extends to the T22 encounter
- Three orbit trim maneuvers (OTMs) between the Titan encounters.
- 16 days between encounters
- 16 day orbit period

Titan 21 to Titan 22

16 day resonate transfer

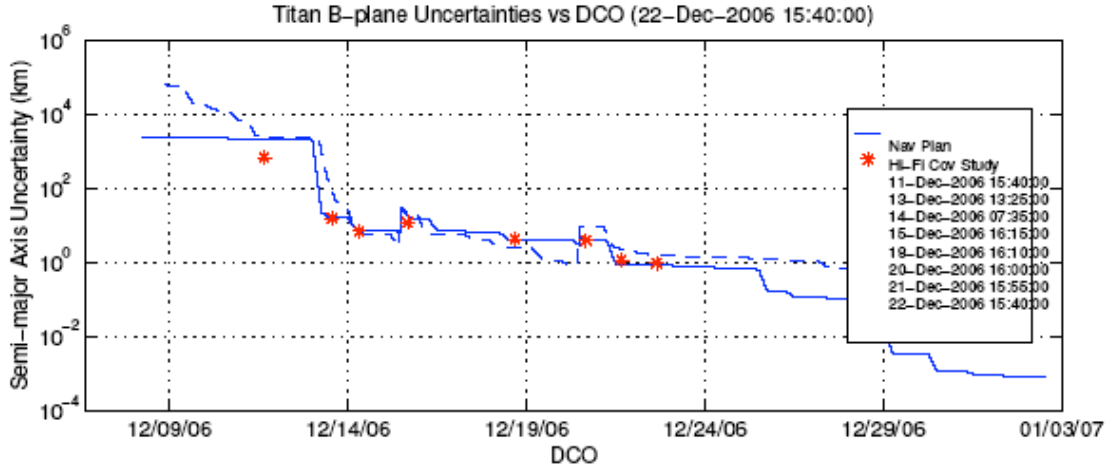


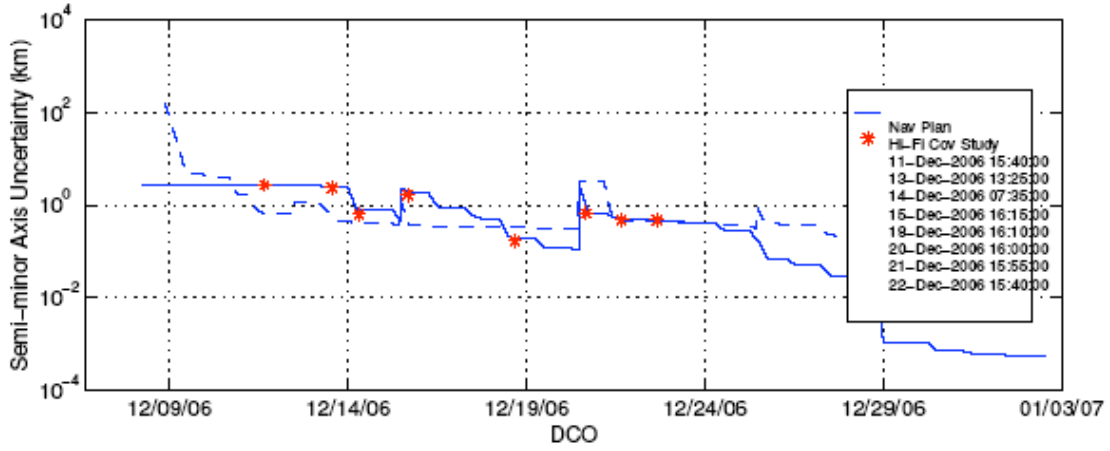
Arc Epoch (ET)	08-DEC-2006 06:00:00	
OTM082 DCO	08-DEC-2006 16:45:00	
Saturn Apo	08-DEC-2006 21:02:17	
OTM082	09-DEC-2006 12:32:00	T21 - 3 days
Titan-21	12-DEC-2006 11:41:31	1000 km, RCS
RWA bias	14-DEC-2006 02:59:10	
OTM083 DCO	14-DEC-2006 07:40:00	
Saturn Peri	15-DEC-2006 00:02:00	
OTM083	15-DEC-2006 12:03:00	T21 + 3 days
RWA bias	18-DEC-2006 10:21:10	
OTM084 DCO	19-DEC-2006 16:00:00	
OTM084	20-DEC-2006 11:48:00	Apo Mvr
Saturn Apo	22-DEC-2006 23:11:47	
OTM085 DCO	23-DEC-2006 08:15:00	
OTM085	25-DEC-2006 11:34:00	T22 - 3 days
Titan-22	28-DEC-2006 10:05:22	1300 km, RWA
RWA bias	29-DEC-2006 04:50:10	
Arc End (ET)	30-DEC-2006 12:00:00	

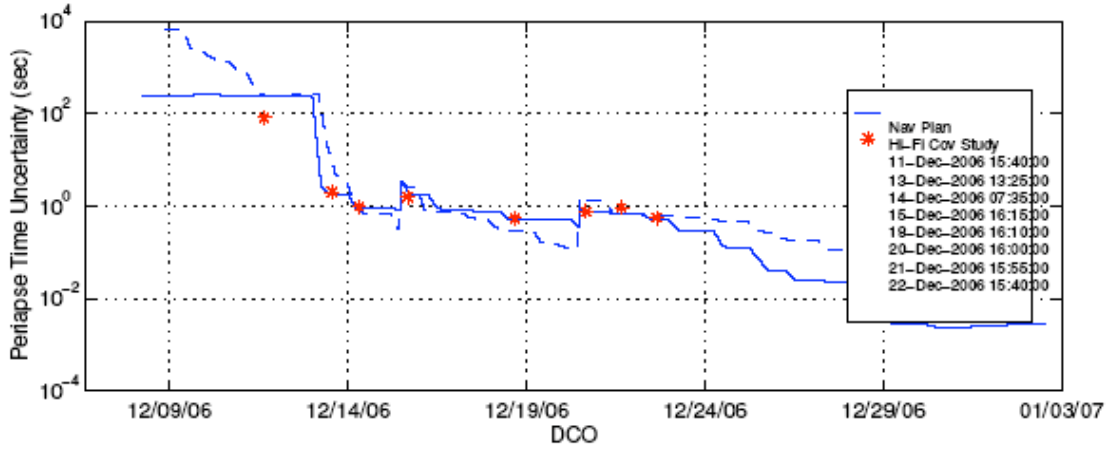


T22 Uncertainties at OTM DCOs (not including future maneuver uncertainties)

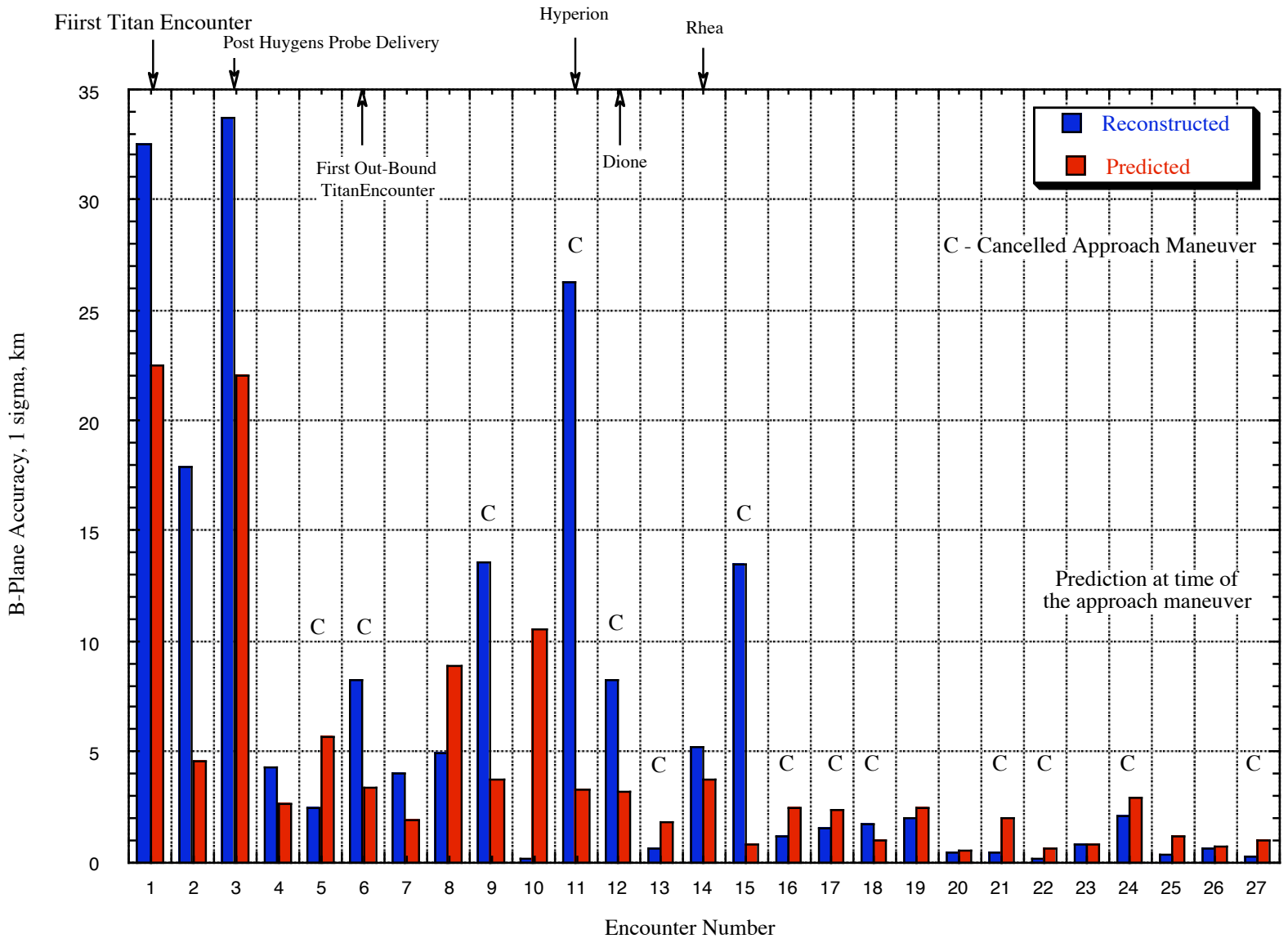
OTM	Name	DCO (UTC)	B·R (km)	B·T (km)	Altitude (km)	TCA (s)	SMAA (km)	SMIA (km)
OTM083	T21 CU	14-DEC 07:40:00	3.4	6.4	2.9	0.9	7.2	0.8
OTM084	Apo	19-DEC 16:00:00	2.3	3.5	2.0	0.5	4.2	0.2
OTM085	T22 APR	23-DEC 08:15:00	0.5	0.7	0.5	0.3	0.7	0.4


Titan B–plane Uncertainties vs DCO (22–Dec–2006 15:40:00)
Semi–major Axis Uncertainty (km)
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
12/09/06
12/14/06
12/19/06
12/24/06
12/29/06
01/03/07
DCO
Nav Plan
HI–FI Cov Study
11–Dec–2006 15:40:00
13–Dec–2006 13:25:00
14–Dec–2006 07:35:00
15–Dec–2006 16:15:00
19–Dec–2006 16:10:00
20–Dec–2006 16:00:00
21–Dec–2006 15:55:00
22–Dec–2006 15:40:00

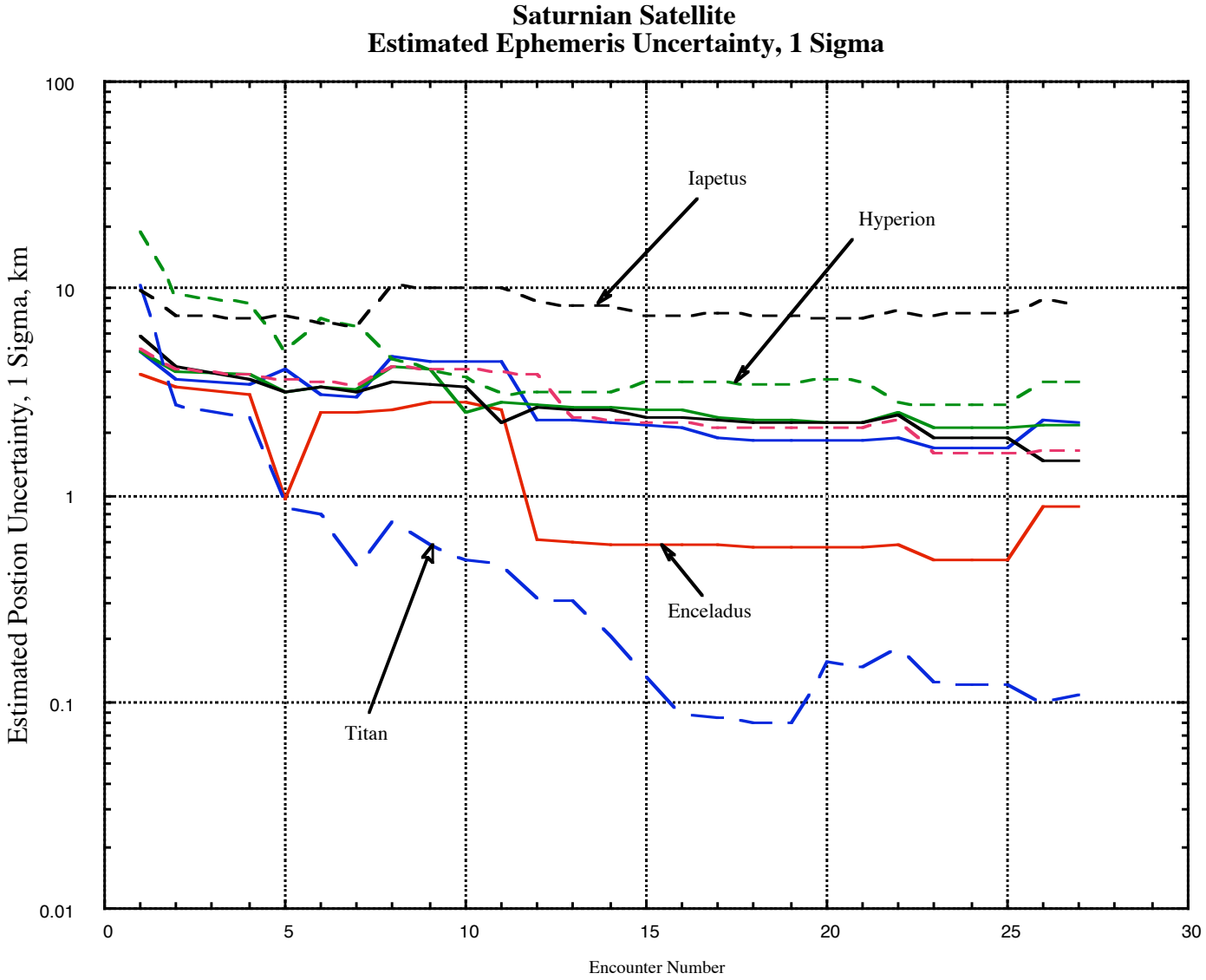

Semi–minor Axis Uncertainty (km)
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
12/09/06
12/14/06
12/19/06
12/24/06
12/29/06
01/03/07
DCO
Nav Plan
HI–FI Cov Study
11–Dec–2006 15:40:00
13–Dec–2006 13:25:00
14–Dec–2006 07:35:00
15–Dec–2006 16:15:00
19–Dec–2006 16:10:00
20–Dec–2006 16:00:00
21–Dec–2006 15:55:00
22–Dec–2006 15:40:00


Periapse Time Uncertainty (sec)
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
12/09/06
12/14/06
12/19/06
12/24/06
12/29/06
01/03/07
DCO
Nav Plan
HI–FI Cov Study
11–Dec–2006 15:40:00
13–Dec–2006 13:25:00
14–Dec–2006 07:35:00
15–Dec–2006 16:15:00
19–Dec–2006 16:10:00
20–Dec–2006 16:00:00
21–Dec–2006 15:55:00
22–Dec–2006 15:40:00

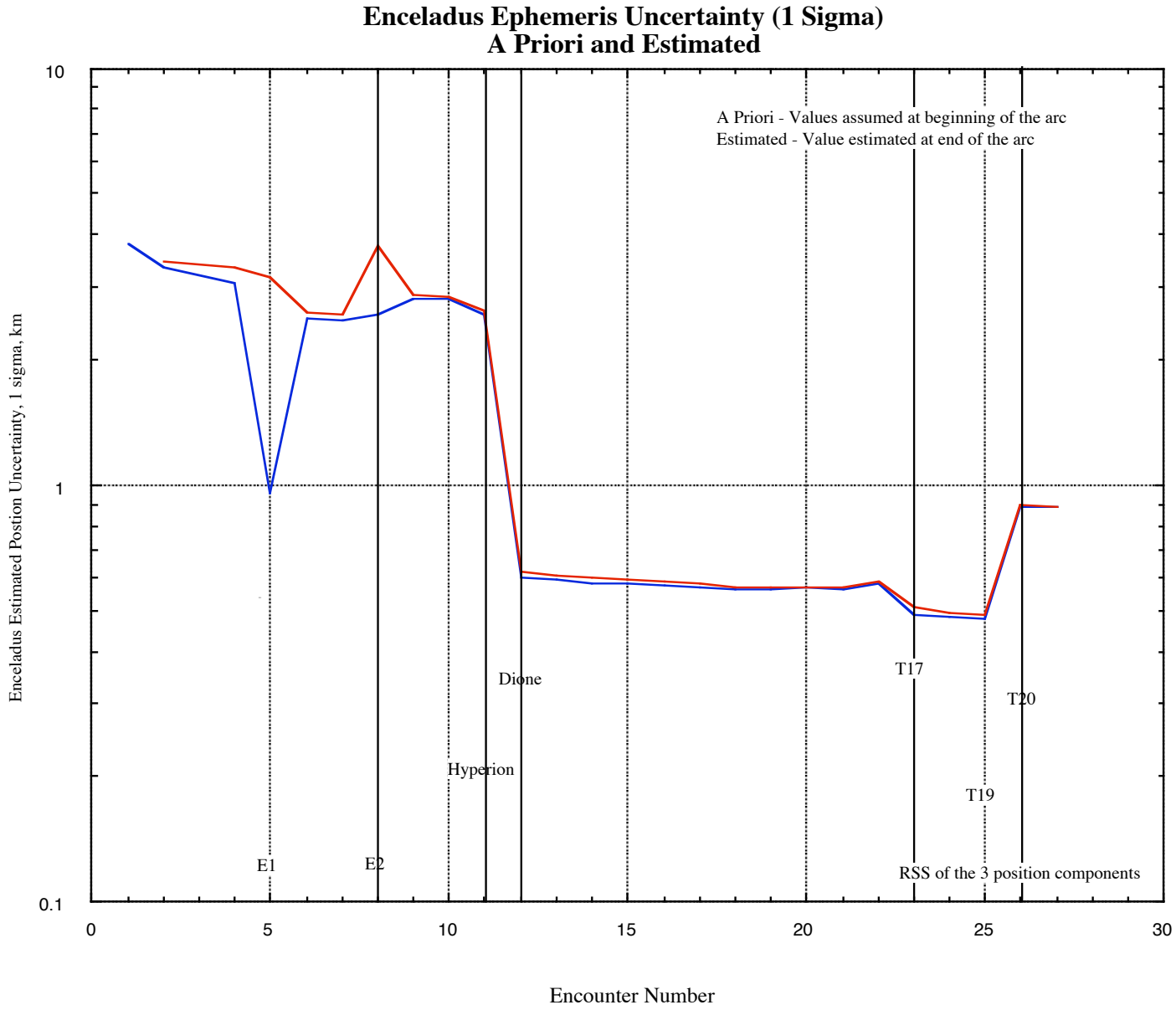
Overall Results


Predicted and Reconstructed B-Plane Error (1 Sigma)
Relative to Reference Trajectory
Fiirst Titan Encounter
Post Huygens Probe Delivery
Hyperion
Rhea
First Out-Bound
TitanEncounter
Dione
Reconstructed
Predicted
C - Cancelled Approach Maneuver
Prediction at time of
the approach maneuver
B-Plane Accuracy, 1 sigma, km
Encounter Number
0
5
10
15
20
25
30
35
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27

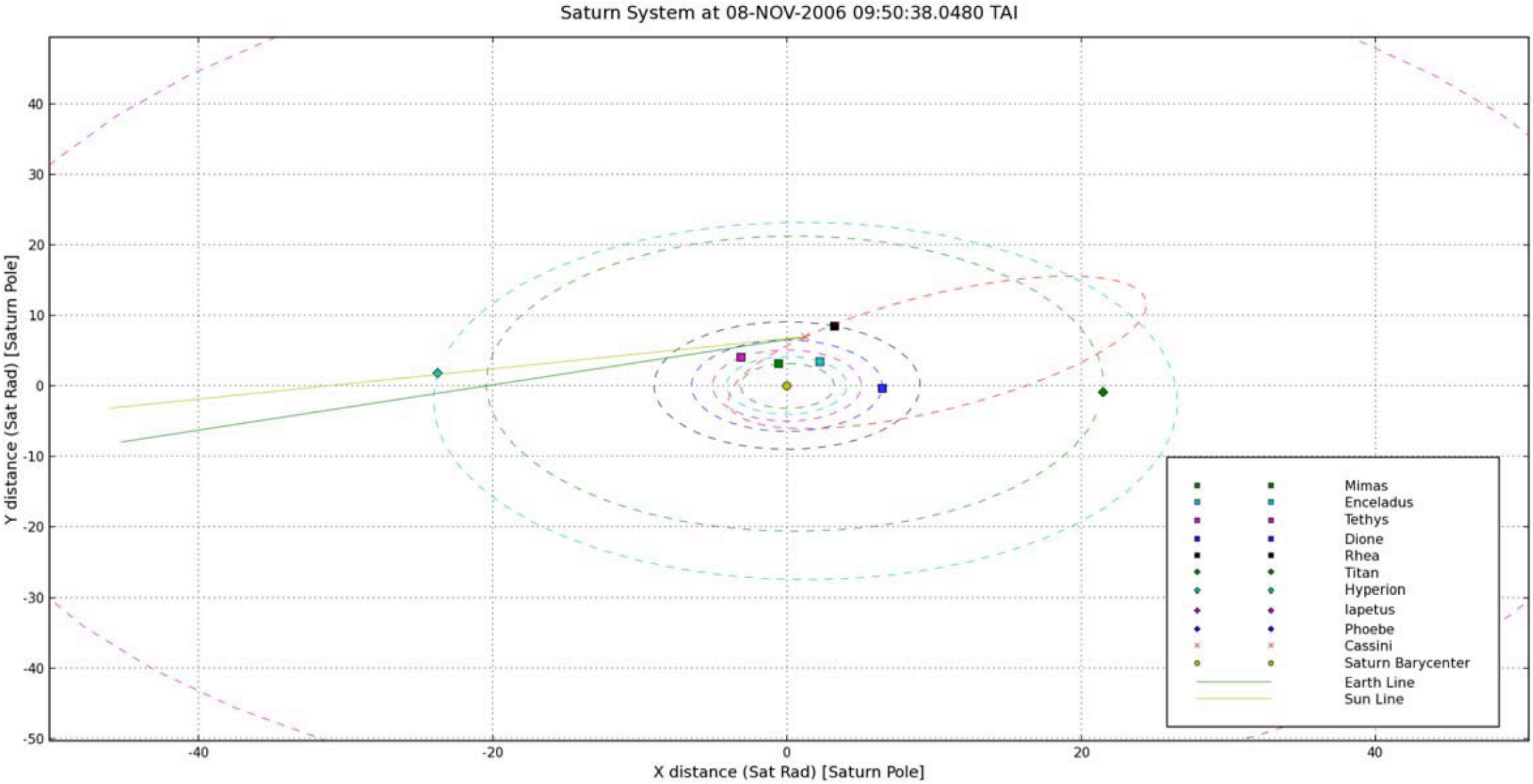
Satellite Ephemeris Accuracy

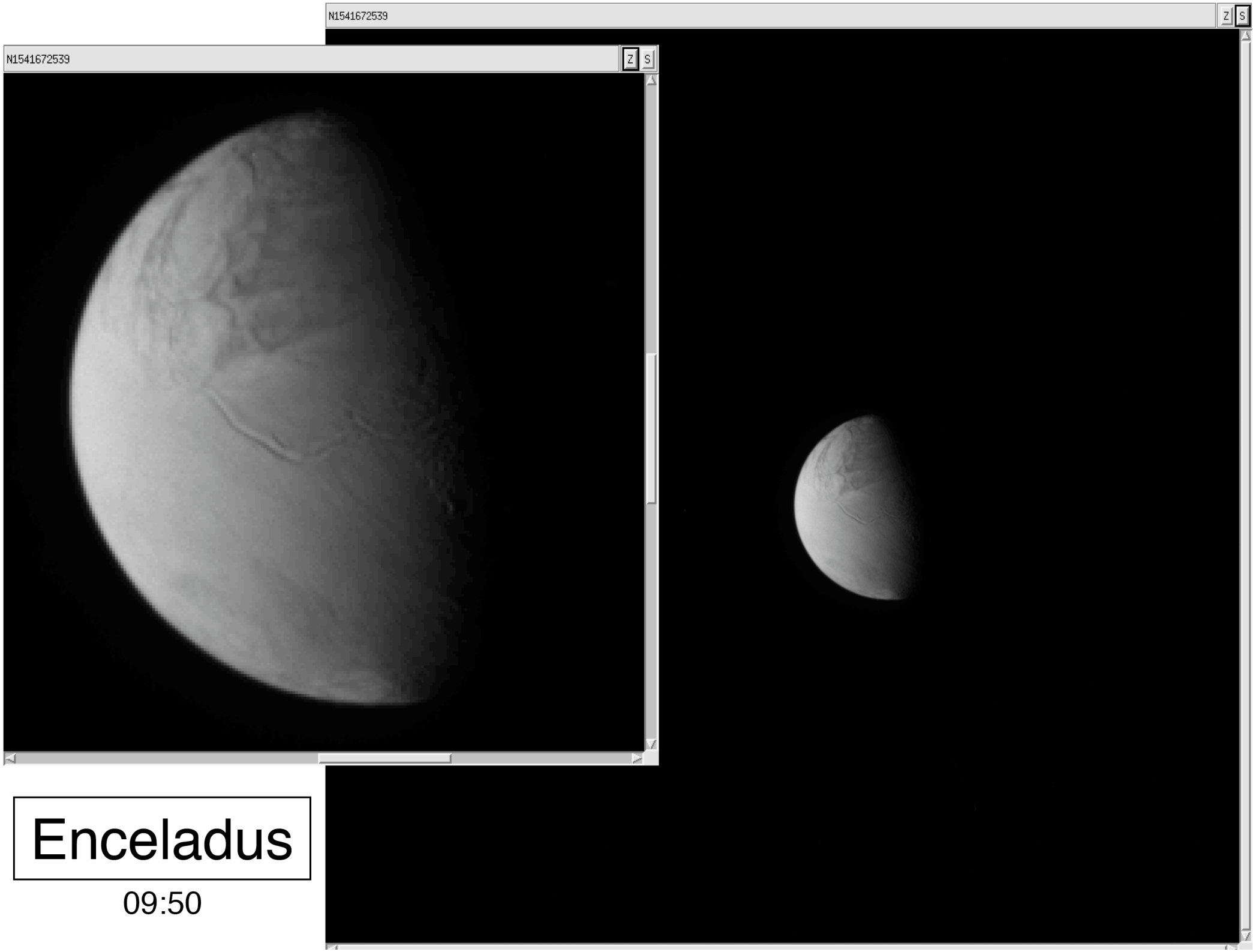


Satellite Ephemeris Accuracy



Optical Navigation Geometry

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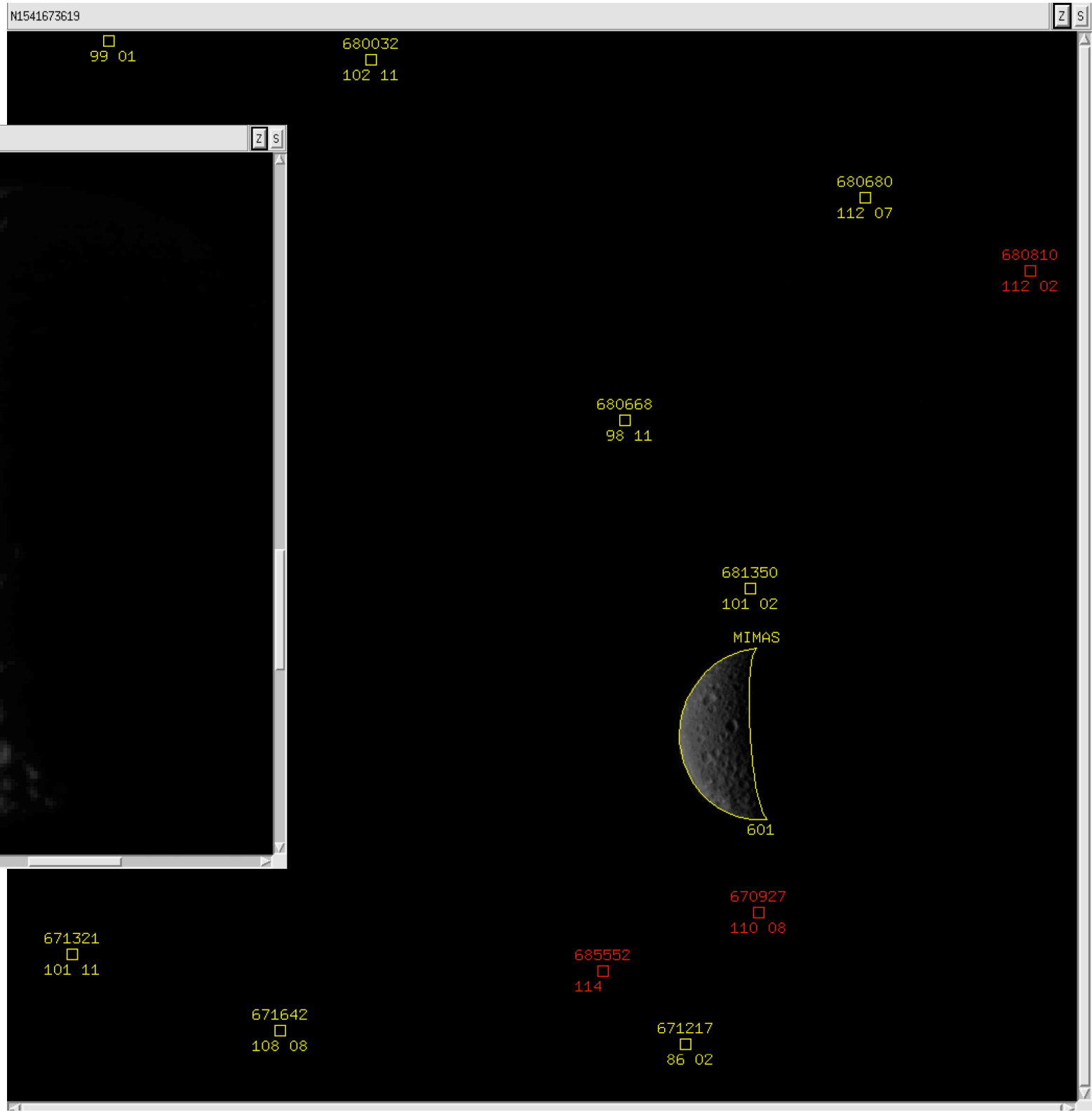
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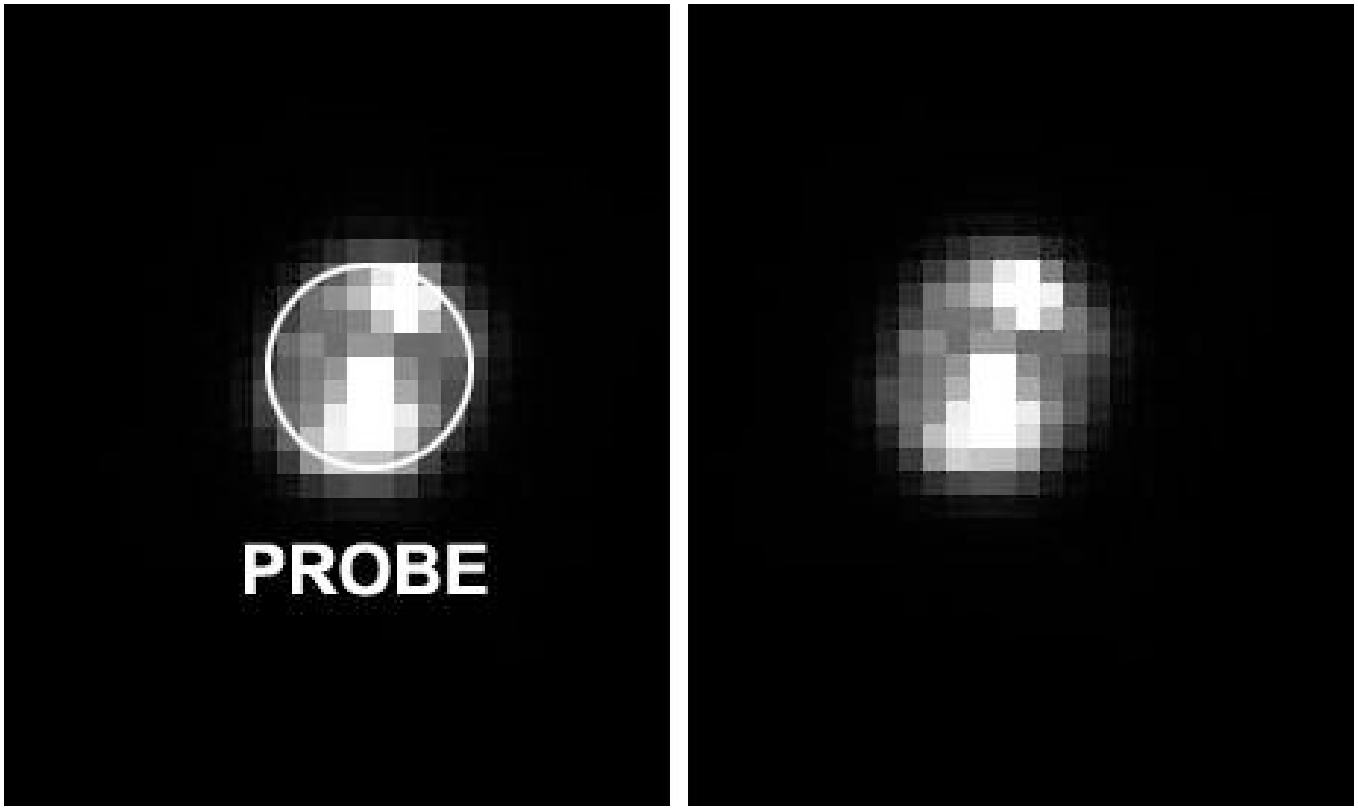
Probe Images



Control

- Typically maneuvers (changes in the spacecraft velocity) are required in order to achieve the objectives of the mission
    - Deterministic maneuvers
        - When the "free fall" trajectory does not go the desired terminal condition, a deterministic maneuver will be required,
    - Statistical maneuvers
        - Correcting errors in the trajectory resulting from:
            - Launch vehicles injection errors
            - Errors in the estimate of the orbit
            - Errors in the execution of a previous maneuver
            - Errors in a flyby

Mathematics

Using state transition matrix the state (position and velocity) at the time t_i is related to the initial state, t_0 by :

$$s(t_i) = \begin{bmatrix} x(t_f) \\ v(t_f) \end{bmatrix} = \Phi(t_i, t_0) s(t_0) = \Phi(t_i, t_0) \begin{bmatrix} x(t_0) \\ v(t_0) \end{bmatrix}$$

The influence of manuvers on the final state is

$$s(t_f) = \Phi(t_f, t_0) s(t_0) + \sum_i \Phi(t_f, t_i) \begin{bmatrix} 0 \\ \Delta V_i \end{bmatrix}$$

Therefore the change in the final state is;

$$\Delta s(t_f) = s(t_f) - \Phi(t_f, t_0) s(t_0) = \sum_i \Phi(t_f, t_i) \begin{bmatrix} 0 \\ \Delta V_i \end{bmatrix} = \sum_i \begin{pmatrix} \Phi_{x,x} & \Phi_{x,v} \\ \Phi_{v,x} & \Phi_{v,v} \end{pmatrix} \begin{bmatrix} 0 \\ \Delta V_i \end{bmatrix}$$

A single maneuver (with 3 parameters) can only control 3 parameters at the final time

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A single maneuver (with 3 parameters) can only control 3 parameters at the final time

Mathematics (Continued)

If the objective is to control the position at the final time, then the required maneuver is:

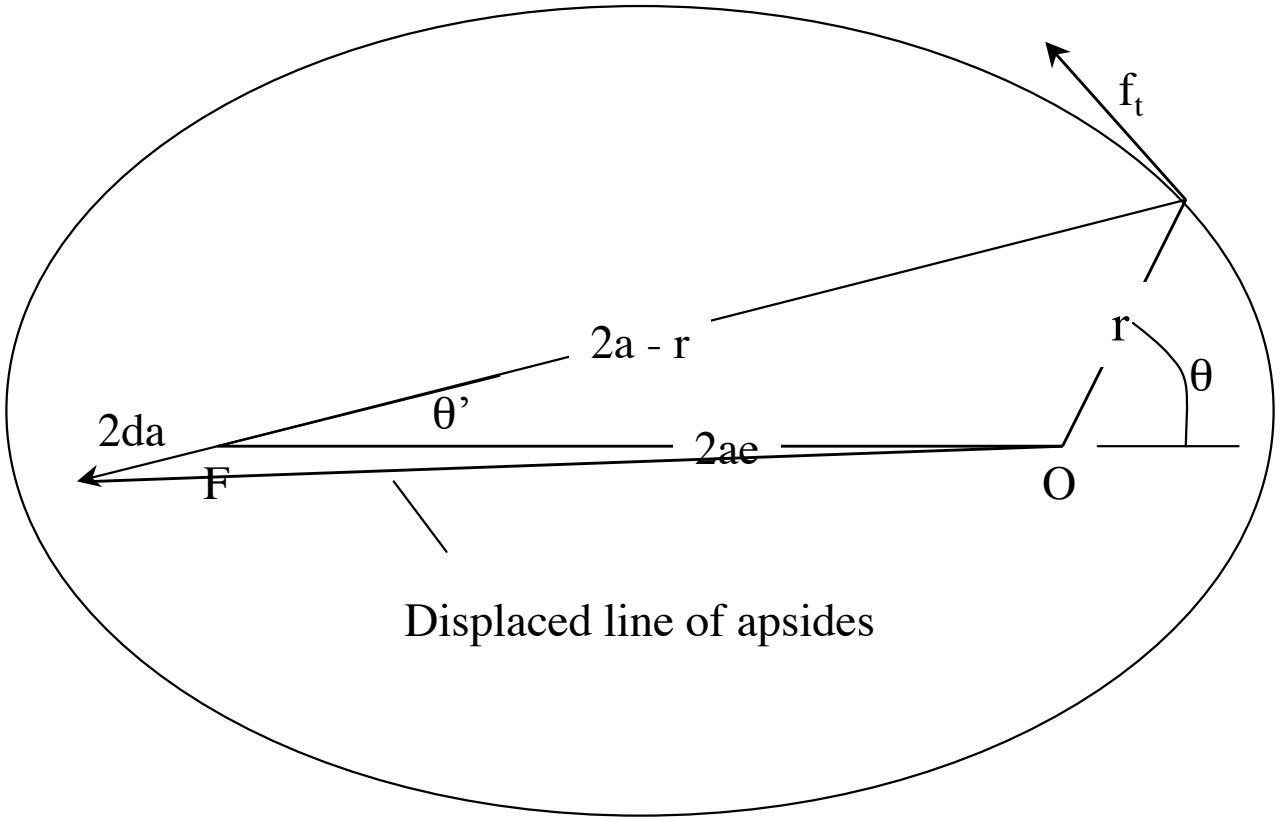
$$\Delta V = \Phi_{x,v}^{-1}(t_f, t_i)\Delta x(t_f)$$

This is the simplest maneuver strategy used in interplanetary navigation. The magnitude of the maneuver is a function of both the desired correction and the time of the maneuver.

If the requirement is control more than 3 components of the final state then multiple maneuvers will be required.

For example, if the requirement is to rendezvous with the target (match position and velocity) then a minimum of 2 maneuvers will be required.

Orbit Control - Variation of Parameters - Tangential Impulse



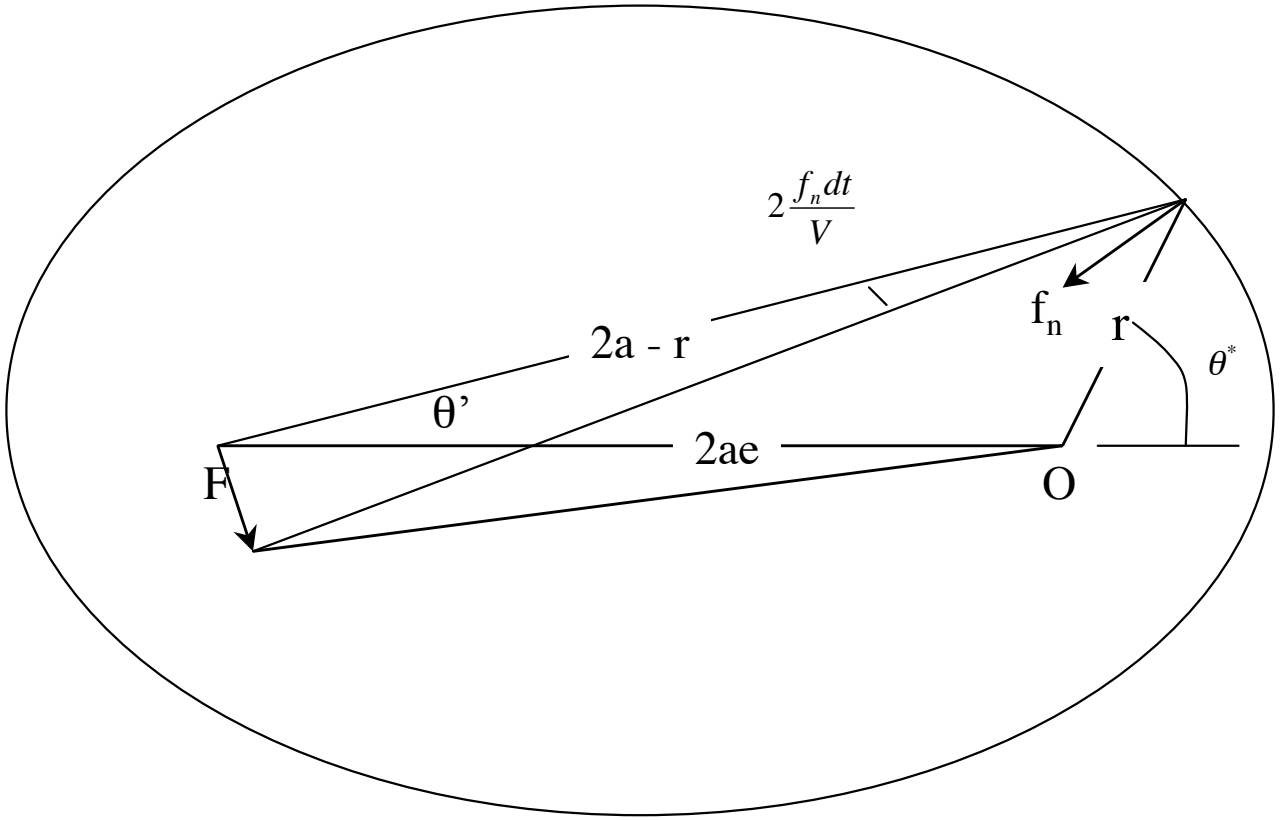
Tangential Impulse, f_t

$$Vf_t dt = \frac{\mu}{2a^2} da$$

Vacant focus, F, is displaced from the instaneous position by 2da

Orbit changes both energy and orientation

Orbit Control - Variation of Parameters - Radial Impulse



Radial Impulse - $f_n dt$

*Le*aves semi - major axis unchanged

Changes direction of the tangent by $\frac{f_n dt}{V}$

The direction of the vacant focus changes by $2\frac{f_n dt}{V}$

Orbit Control - Variation of Parameters In-Plane Result

Using geometry and orbit relationships results in

$$\dot{e} = \frac{r}{h}\left\{\sin\left(\theta^*\right)\left(1+e\cos\left(\theta^*\right)\right)f_n + \left(e+2\cos\left(\theta^*\right)+e\cos^2\left(\theta^*\right)\right)f_t\right\}$$

$$\dot{\omega} = \frac{r}{he}\left\{-\cos\left(\theta^*\right)\left(1+e\cos\left(\theta^*\right)\right)f_n + \sin\left(\theta^*\right)\left(2+e\cos\left(\theta^*\right)\right)f_t\right\}$$

where

h - angular momentum

r - radius

e - eccentricity

ω - argument of periapsis

θ^* - true anomaly

Propulsion Systems

- Bi-Propellant
  - High thrust and efficiency
  - Cassini
    - 440 Newton thrust, Isp about 305 sec
    - Nitrogen tetroxide (N_2O_4), mono-methyl-hydrazine($N_2H_3CH_3$)
    - Used for maneuvers greater than about .3 m/s
- Mono-Propellant
  - Low thrust, used for attitude control and small maneuvers
  - Cassini
    - About 0.8 Newton thrust, Isp about 220 sec
    - Hydrazine
- Low thrust

Maneuver Objectives

- Orbit maintenance
  - Control 1 or 2 orbit parameter (eccentricity, inclination)
  - Geo-stationary orbits
- Orbit transfer
  - Control 4 orbit parameters (eccentricity vector and inclination vector)
  - Transfer from low Earth orbit to a high Earth orbit
- Rendezvous
  - Control all 6 orbit parameter at specific time
  - Rendezvous of shuttle with Space Station
- Flybys
  - Flyby from one satellite encounter of another (3 parameters at a specific time)

Maneuver Objectives

- The typical maneuver objective is to achieve the mission objective with
  - Maximum accuracy
  - Minimum propellant

- Accuracy depends upon
  - Knowledge of the state and the target at the time of the maneuver
  - Accuracy of the execution of the maneuver
  - Error growth characteristics

- Minimum Propellant
  - Number and location of the maneuvers
  - Orbit Determination and Maneuver accuracy

Maneuver Locations

- Optimal locations
  - All maneuver strategies strive to minimize the propellant cost
  - Typical optimal maneuver locations
    - Energy change - to either change or correct the orbital energy

      The orbital energy is $E = \frac{1}{2}V^2 - \frac{\mu}{r}$

      for an impulsive maneuver, ΔV, the energy change is

      $\Delta E = V * \Delta V + \Delta V^2$

      therefore the optimal location for changing the energy is where the orbit velocity is a maximum.

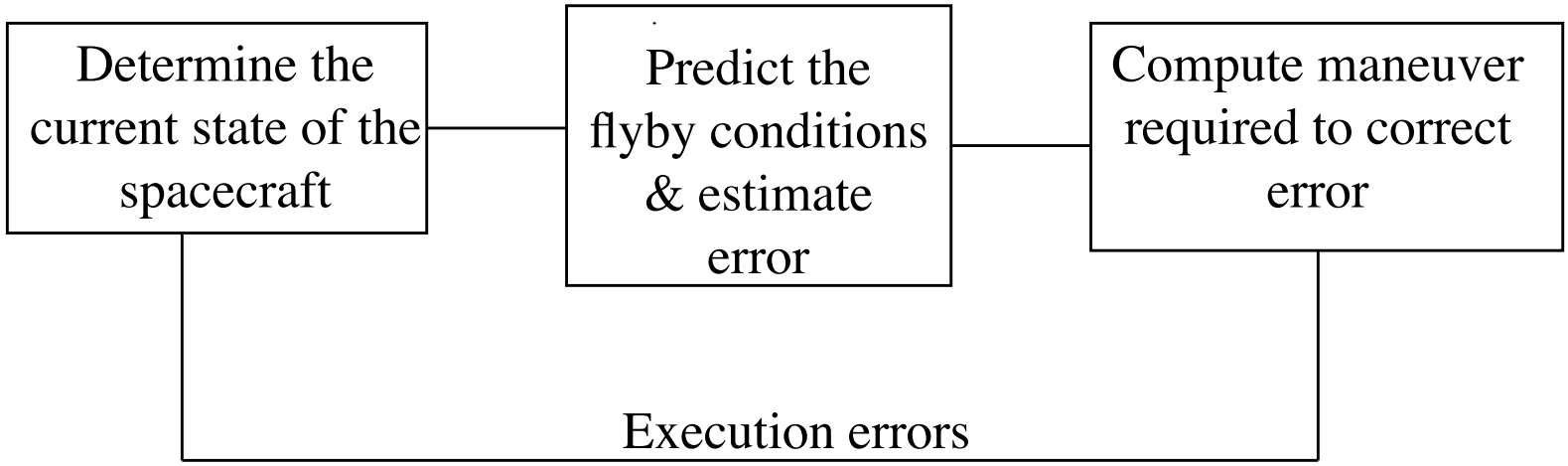
    - Inclination angle

      $$\Delta i = \tan^{-1}\left(\frac{\Delta V}{V}\right)$$

      therefore the maximum change in the relative inclination angle is achieved when the orbital velocity is a minimum

Orbit Determination & Maneuver Interaction

Successful execution of mission like Cassini with many maneuvers requires continued interaction between the orbit determination process and the maneuver process.



Statistical Navigation Analysis

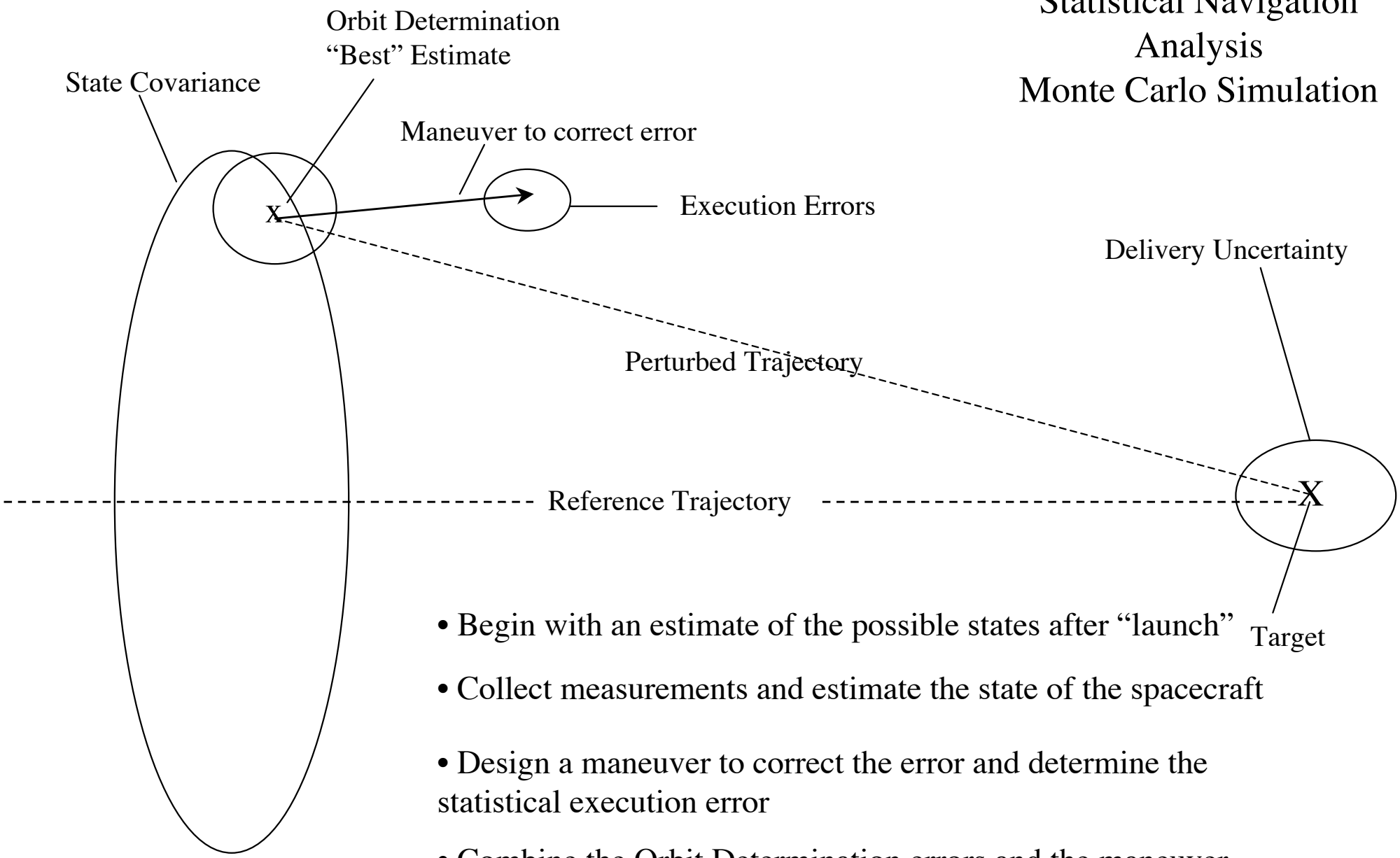
- Prior to the execution of a mission a statistical analysis is conducted in order to obtain information on:
  - Propellant requirements
  - Delivery accuracy
  - Maneuver frequency

- Requires:
  - Simulation of the orbit determination accuracies along the nominal trajectory
  - Selection of the maneuver locations and targeting strategy
  - Generation of maneuver execution error models
  - Mission simulation

Maneuver Execution Errors

- The maneuver execution error model depend upon the design of the spacecraft. Spinning spacecraft and three axis stabilized spacecraft have very different maneuver execution error models.
  - When Cassini uses the main Bi-Propellant engine the maneuver execution errors are predicted to be:
    - Magnitude (3σ)
      - 0.030 m/s fixed
      - 0.6% proportional
    - Pointing (3σ)
      - 0.05 m/s fixed
      - 0.01 radians proportional
  - Thus a 10 m/s maneuver would have
    - statistical magnitude of 10 m/s $\pm$ 0.09 m/s (3σ)
    - statistical pointing error (2 axes) of 0.15 m/s (3σ)

Statistical Navigation Analysis Monte Carlo Simulation



- Begin with an estimate of the possible states after "launch"
- Collect measurements and estimate the state of the spacecraft
- Design a maneuver to correct the error and determine the statistical execution error
- Combine the Orbit Determination errors and the maneuver execution errors to estimate the post maneuver state uncertainty
- Propagate to the target to estimate the delivery accuracy

Statistical Navigation Analysis (Continued)

For a multi-maneuver, multi-encounter mission, like Cassini, requires:

- Specifying the maneuver location and targeting specification for each maneuver
    - Cassini uses both single maneuver targeting and multi-maneuver targeting
- Estimating the orbit determination accuracy prior to each maneuver
    - In practice we compute the predicted accuracy every day
- Conducting a Monte Carlo simulation of the mission
    - Typically we simulate 2,000 separate missions
    - Collect the statistics for each maneuver and each encounter
- Analyze the results
    - Estimate of the propellant required to execute the mission
    - Estimate the delivery accuracy at each target
- Conduct variational studies to determine optimal solutions and sensitivities

Question: Why is a Monte Carlo method used ?

Cassini Tour Maneuver Strategy

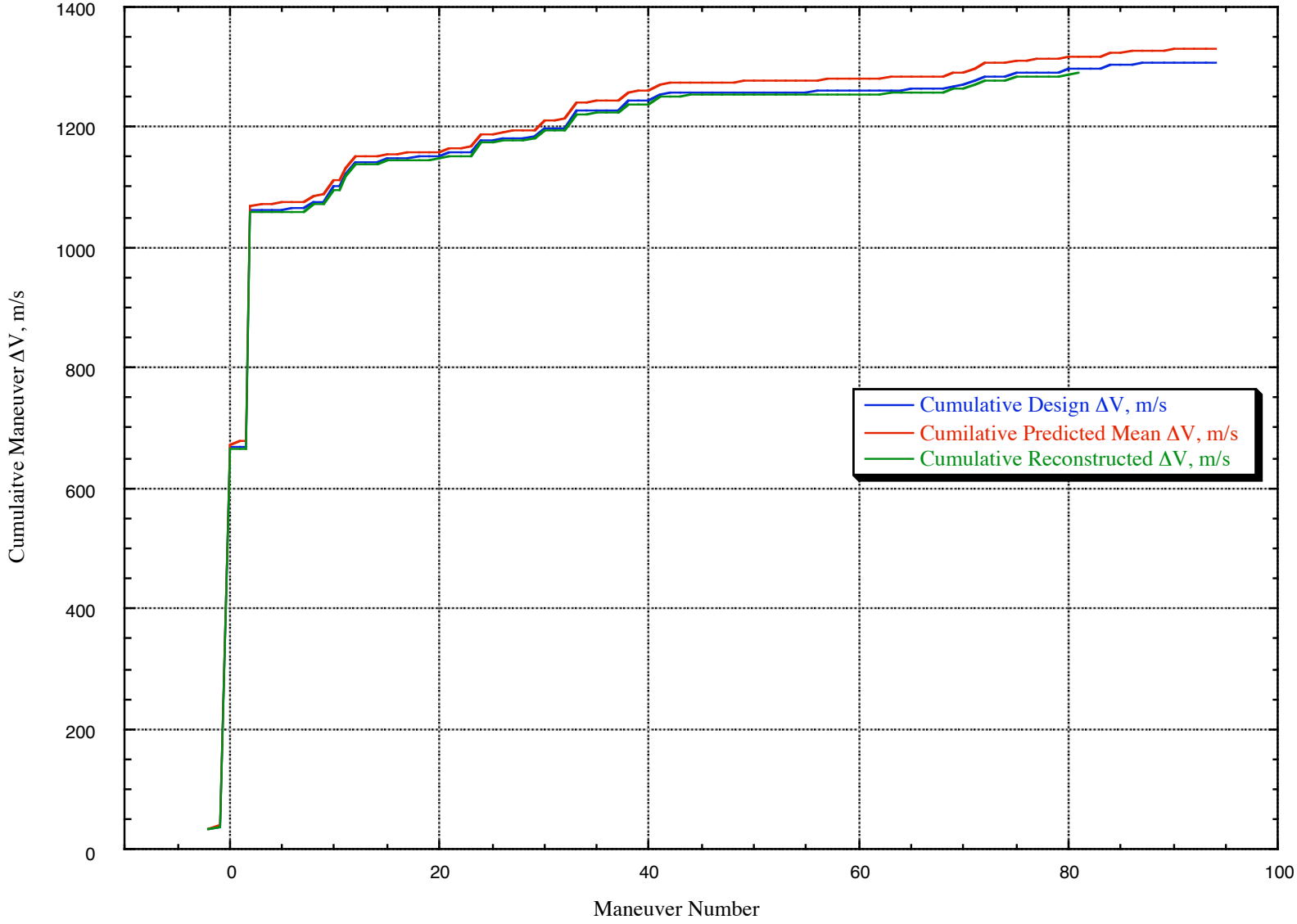
- Cassini Tour Maneuver Strategy
  - Objective is stay on the reference trajectory
    - Science sequences have been planned assuming the reference trajectory and it is very time consuming to replan the science sequences
    - Implemented by controlling the flyby conditions at the targeted flyby
      - Typically 2 B-Plane conditions and the arrival time
  - Typically Cassini schedules three maneuvers between targeted encounters
    - Encounter + 3 days
      - Energy correction maneuver
    - Near Apoapsis
      - Shape correction maneuver
    - Encounter - 3 days
      - Error correction maneuver

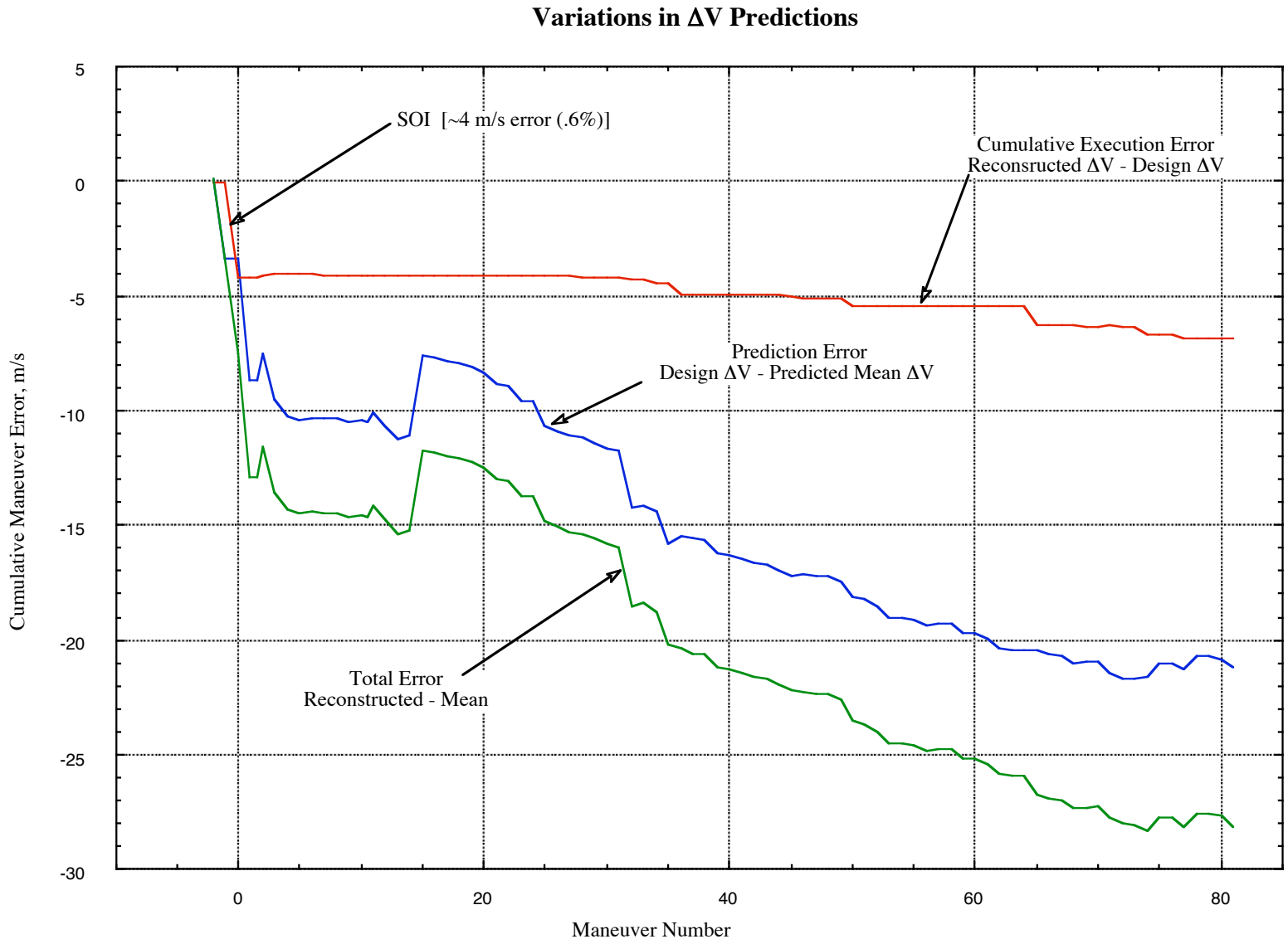
Cassini Tour Maneuver Strategy (Continued)

- Maneuver computations
  - First two maneuver are computed as a pair targeted to the desired flyby location and time.
    - Both maneuvers are required.
  - The approach maneuver achieves an accurate delivery

Results

Cassini DV History
Phoebe Approach Maneuver (27 May 2005) to OTM 93 (7 February 2007)




Variations in ΔV Predictions
SOI [~4 m/s error (.6%)]
Cumulative Execution Error
Reconsructed ΔV - Design ΔV
Prediction Error
Design ΔV - Predicted Mean ΔV
Total Error
Reconstructed - Mean
Cumulative Maneuver Error, m/s
Maneuver Number
5
0
-5
-10
-15
-20
-25
-30
0
20
40
60
80

Cassini Navigation Operation
What are navigators doing during operations?

- Develop detailed predictions of the Orbit Determination accuracy and ΔV for a specified time period (typically includes two targeted encounters)
  - Used to judge the progress of the "actual" orbit determination process

• On a regular basis process the radiometric (doppler and range) data and the optical navigation images.

  • Update the estimates of the spacecraft orbit, the major satellite orbits, satellite masses and other parameters

  • Determine the parameters of the next orbit trim maneuver

• At selected points prior to each scheduled maneuver, a final orbit determination solution and maneuver is selected and sent to the spacecraft team for implementation.

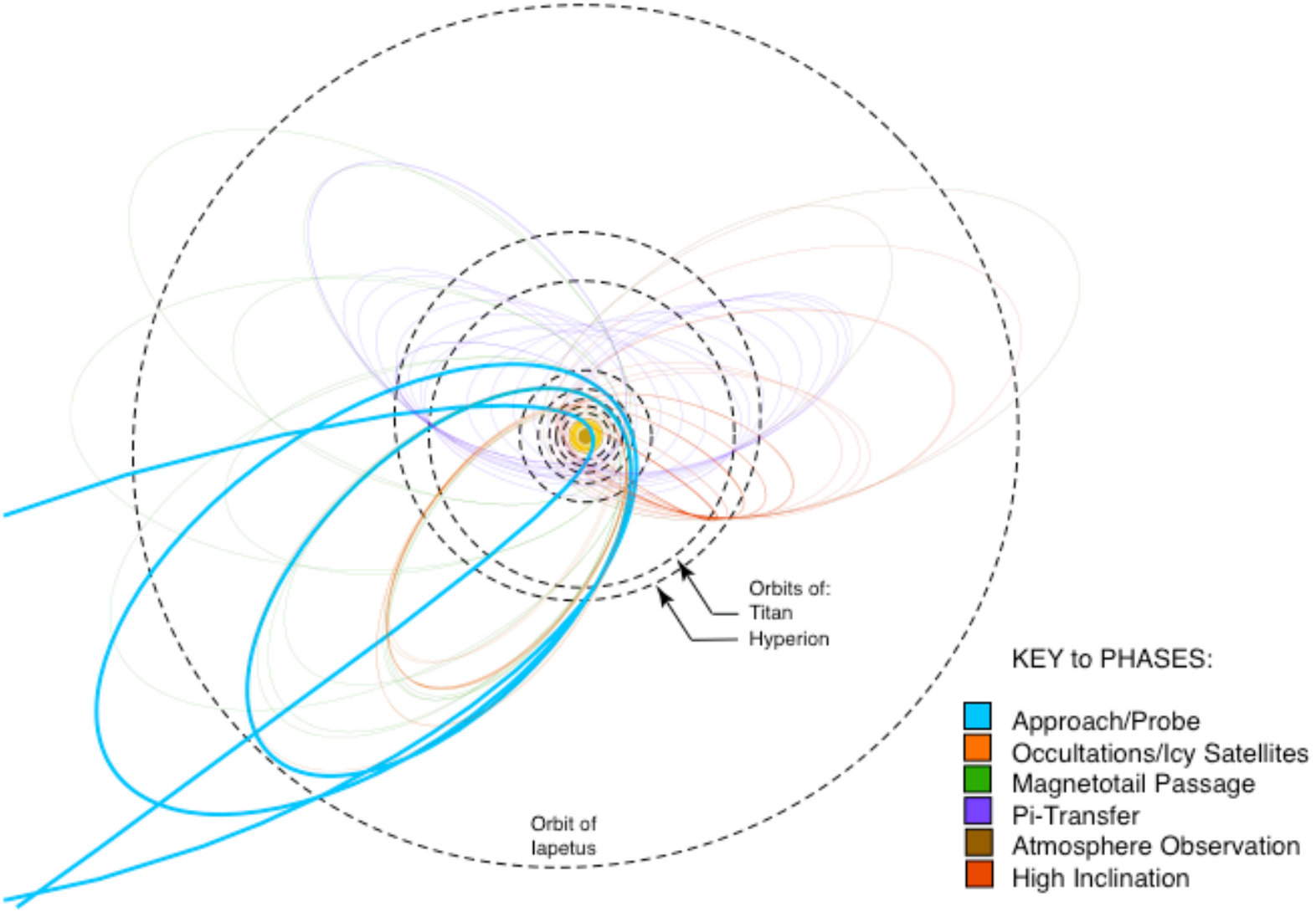
  • In some cases, the maneuver is too small to be implemented and cancellation is recommended.

• We periodically deliver both predicted and reconstructed trajectories

Backup

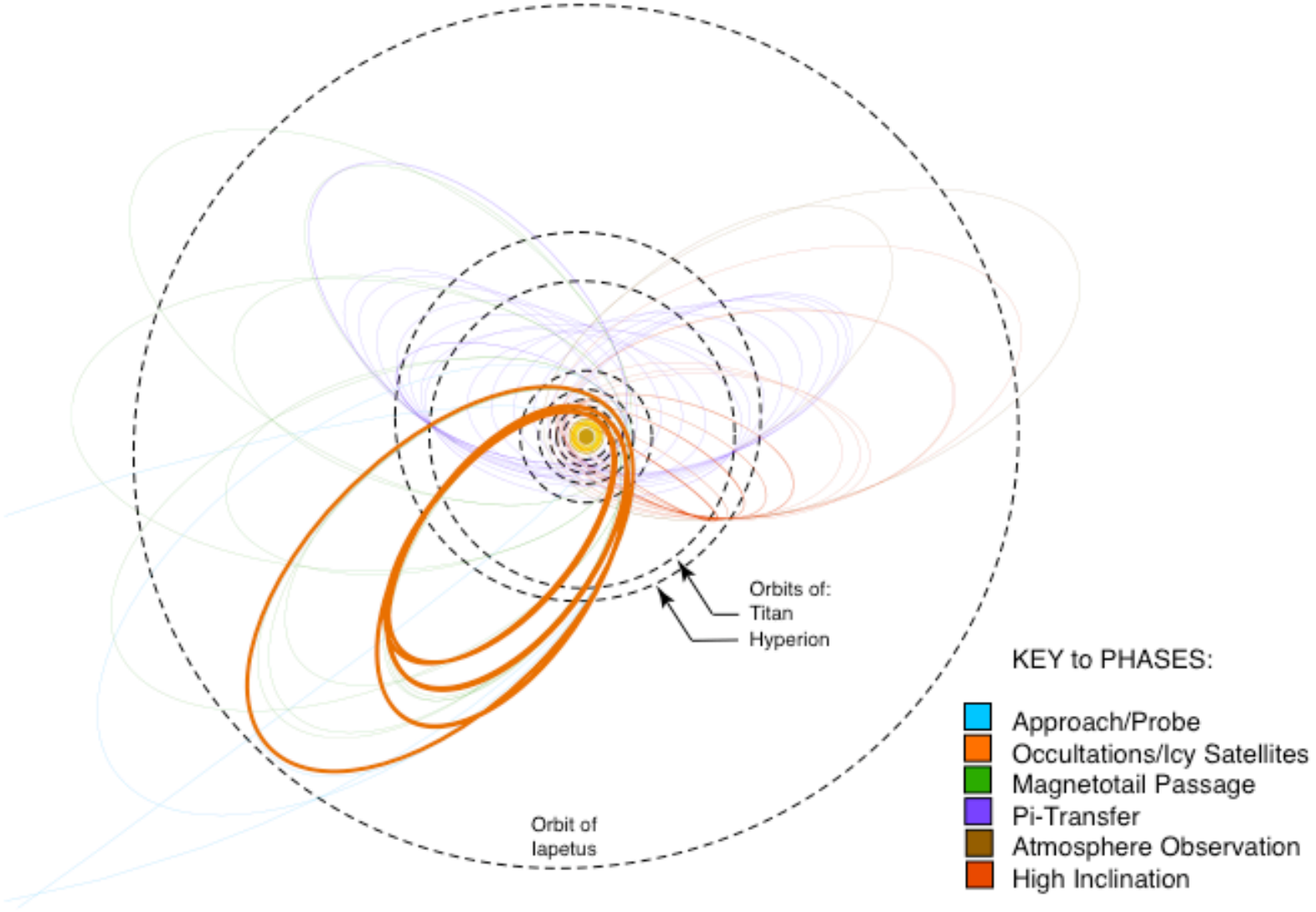
CASSINI MISSION SATELLITE TOUR

Approach/Probe



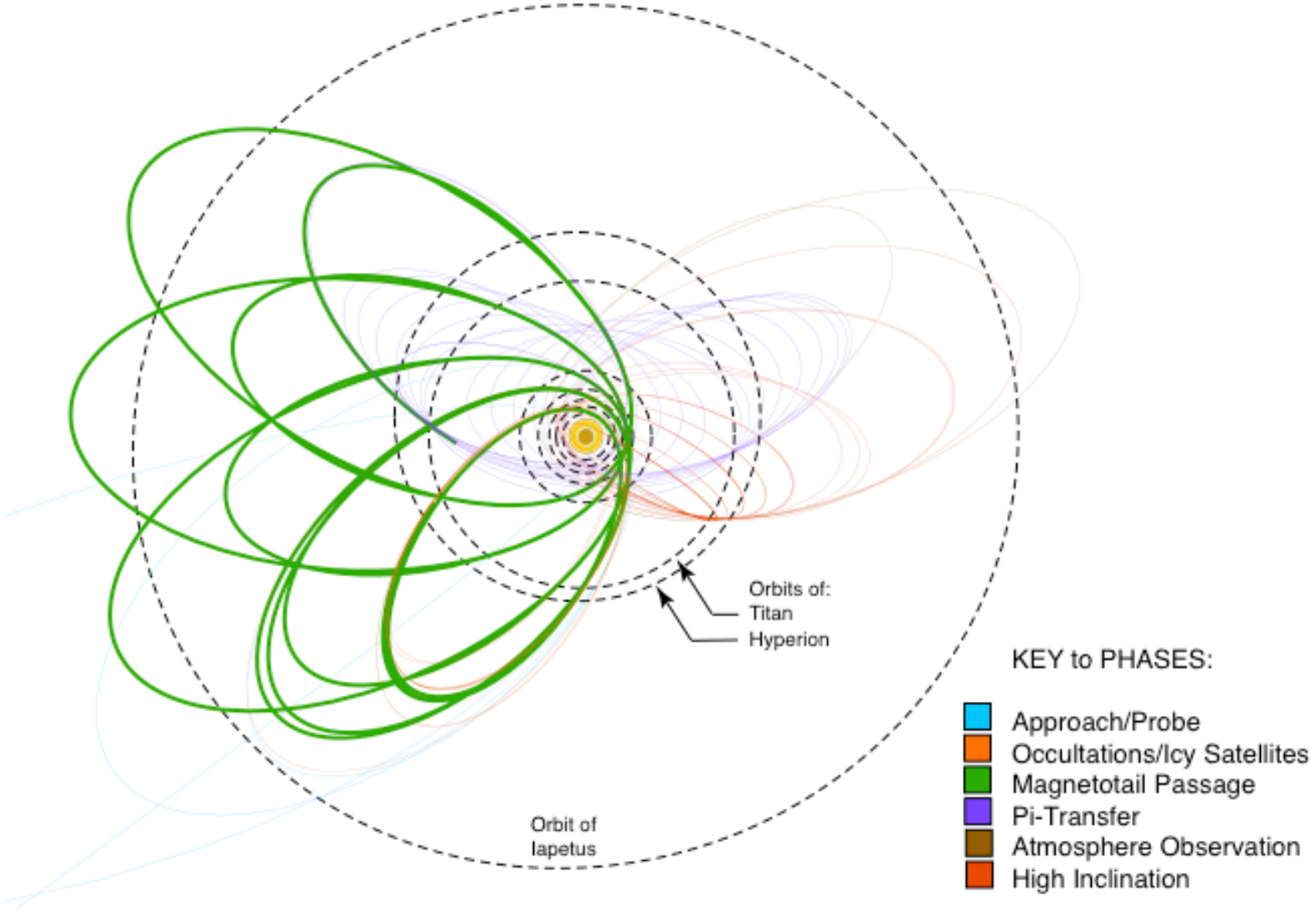
CASSINI MISSION SATELLITE TOUR

Occultations/Icy Satellites



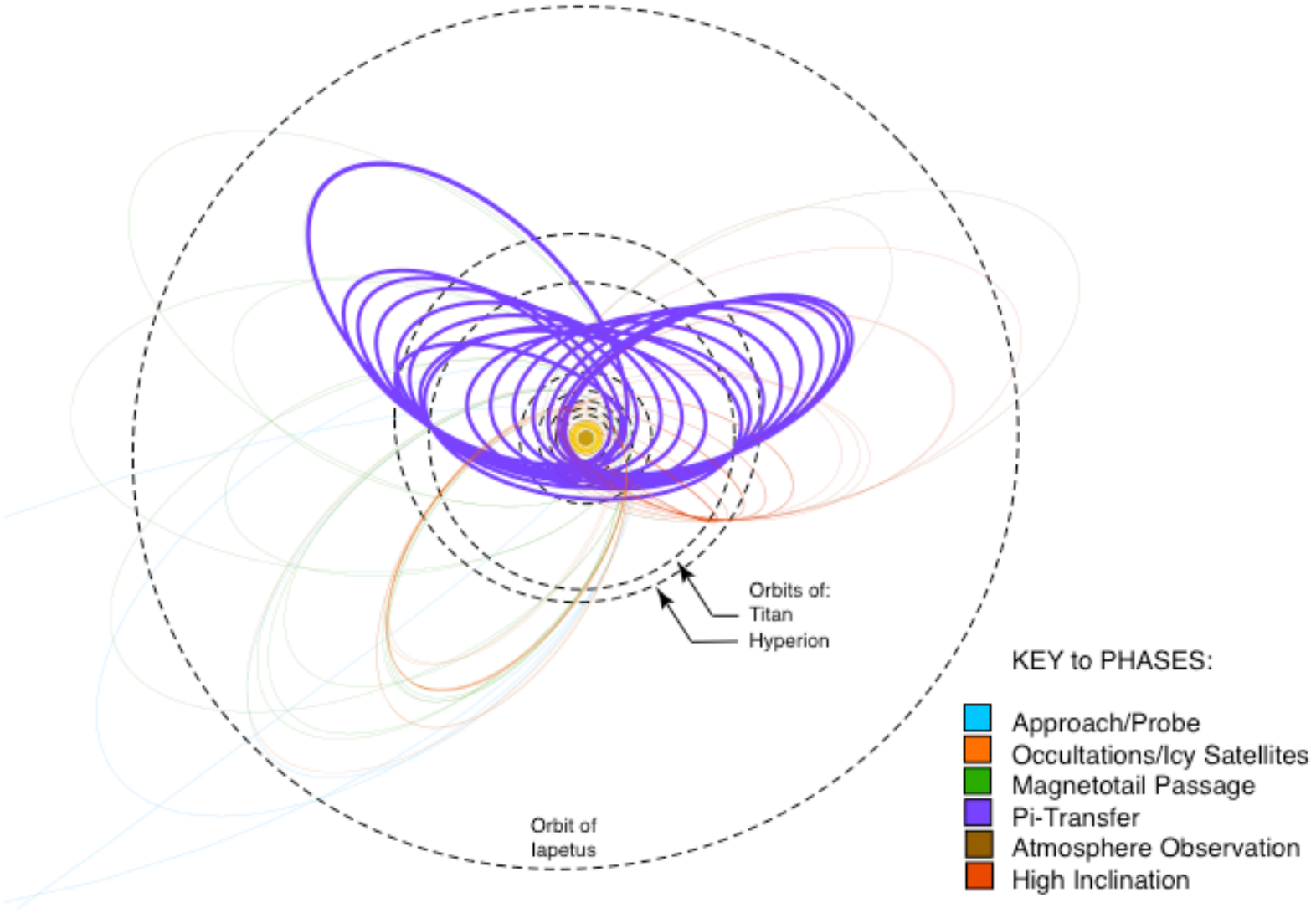
CASSINI MISSION SATELLITE TOUR

Magnetotail Passage



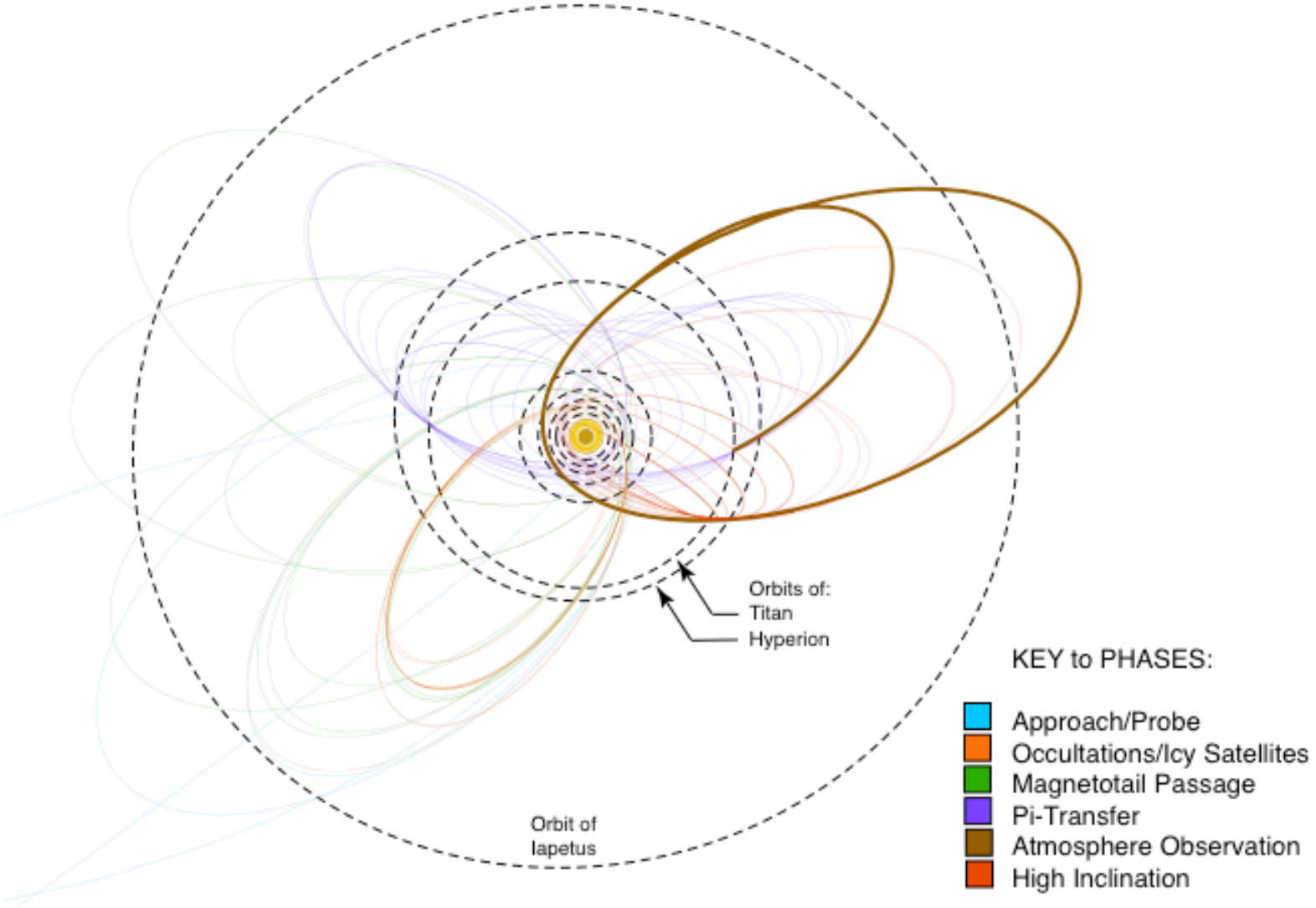
CASSINI MISSION SATELLITE TOUR

Pi-Transfer



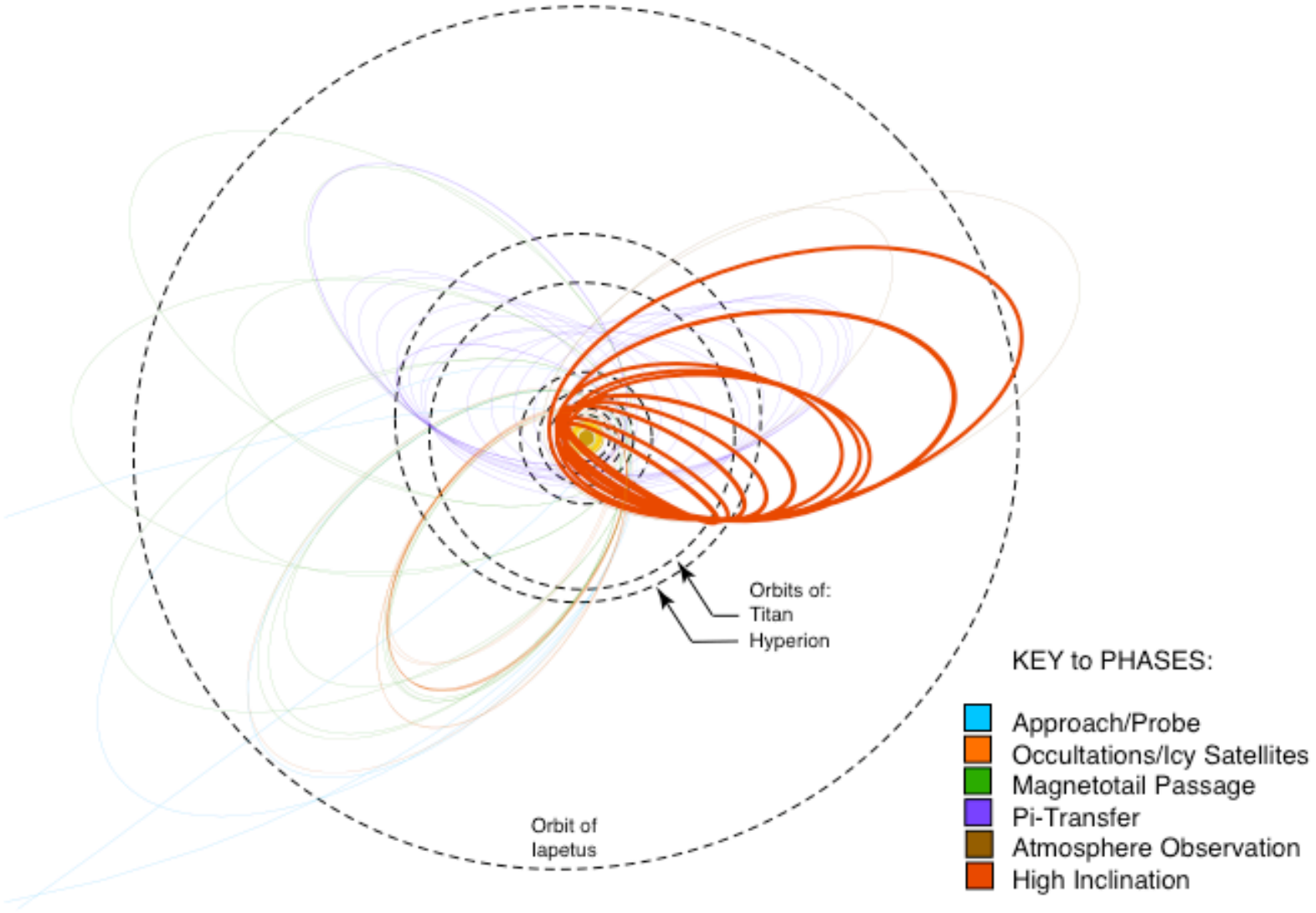
CASSINI MISSION SATELLITE TOUR

Atmospheric Observations

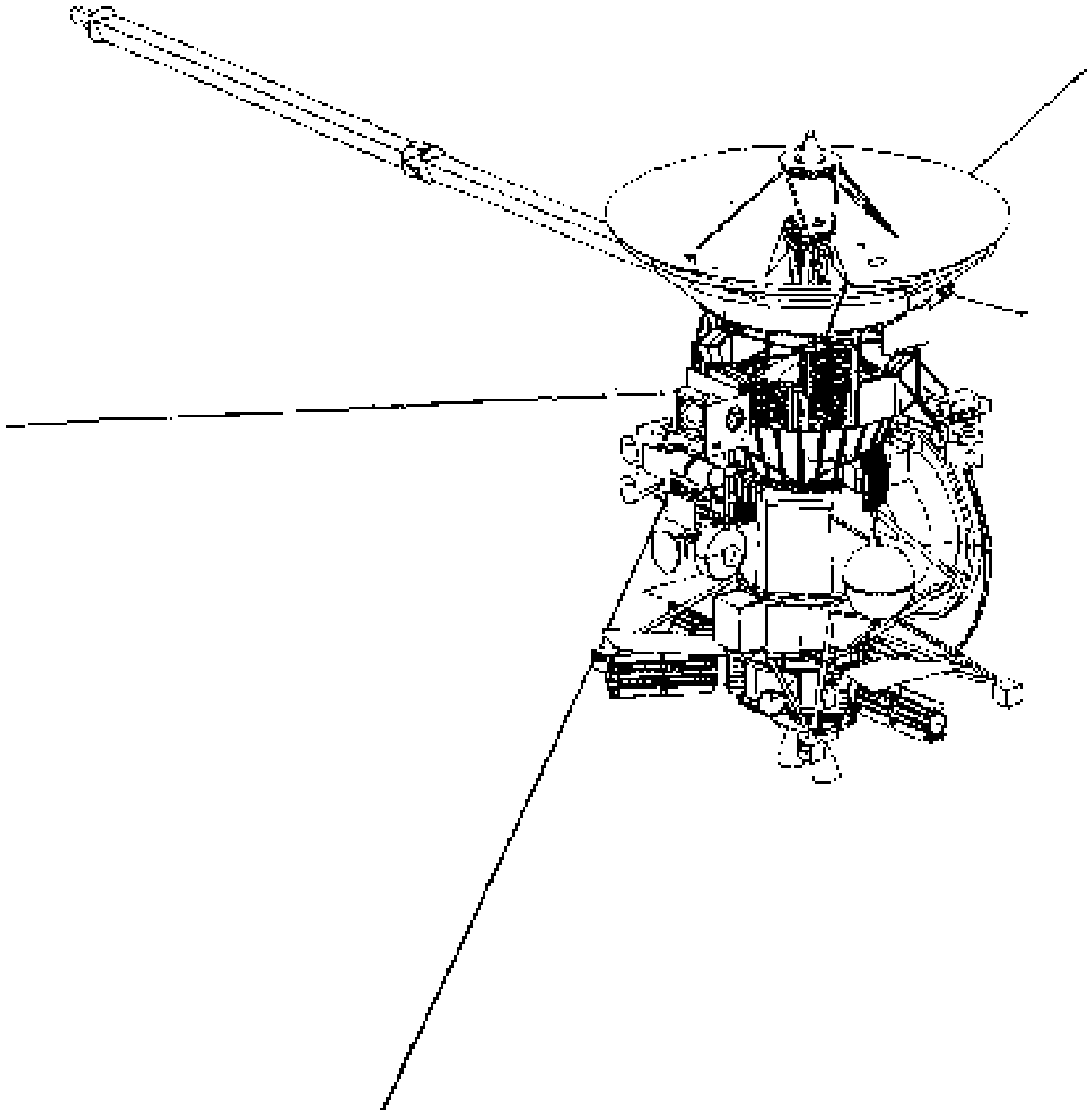


CASSINI MISSION SATELLITE TOUR

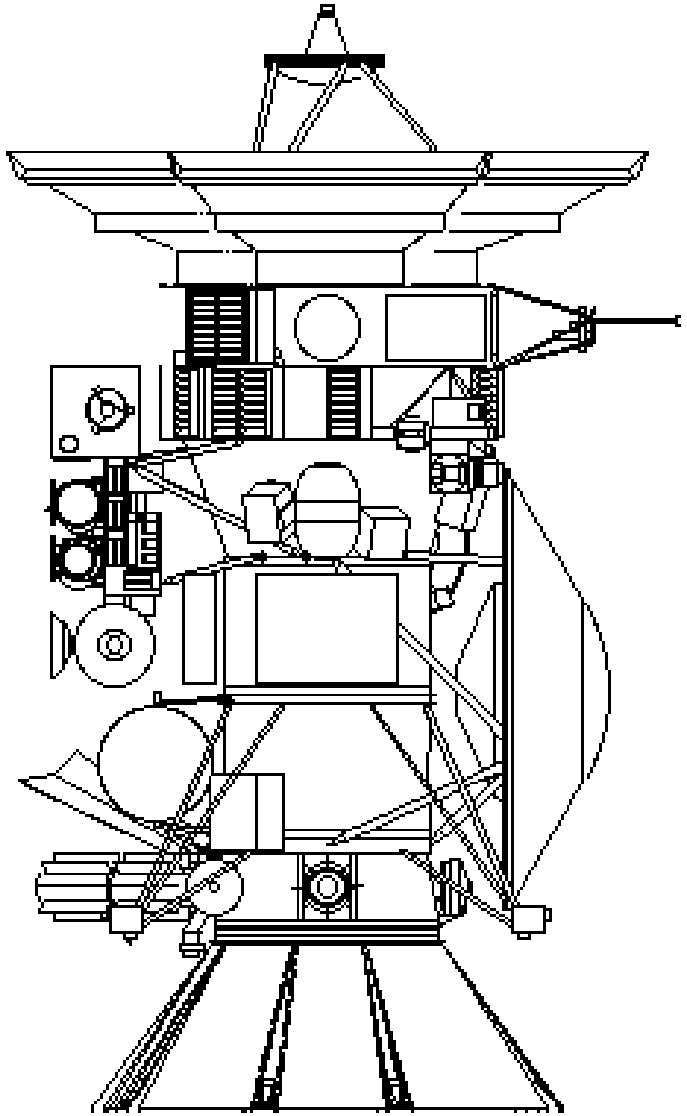
High Inclination

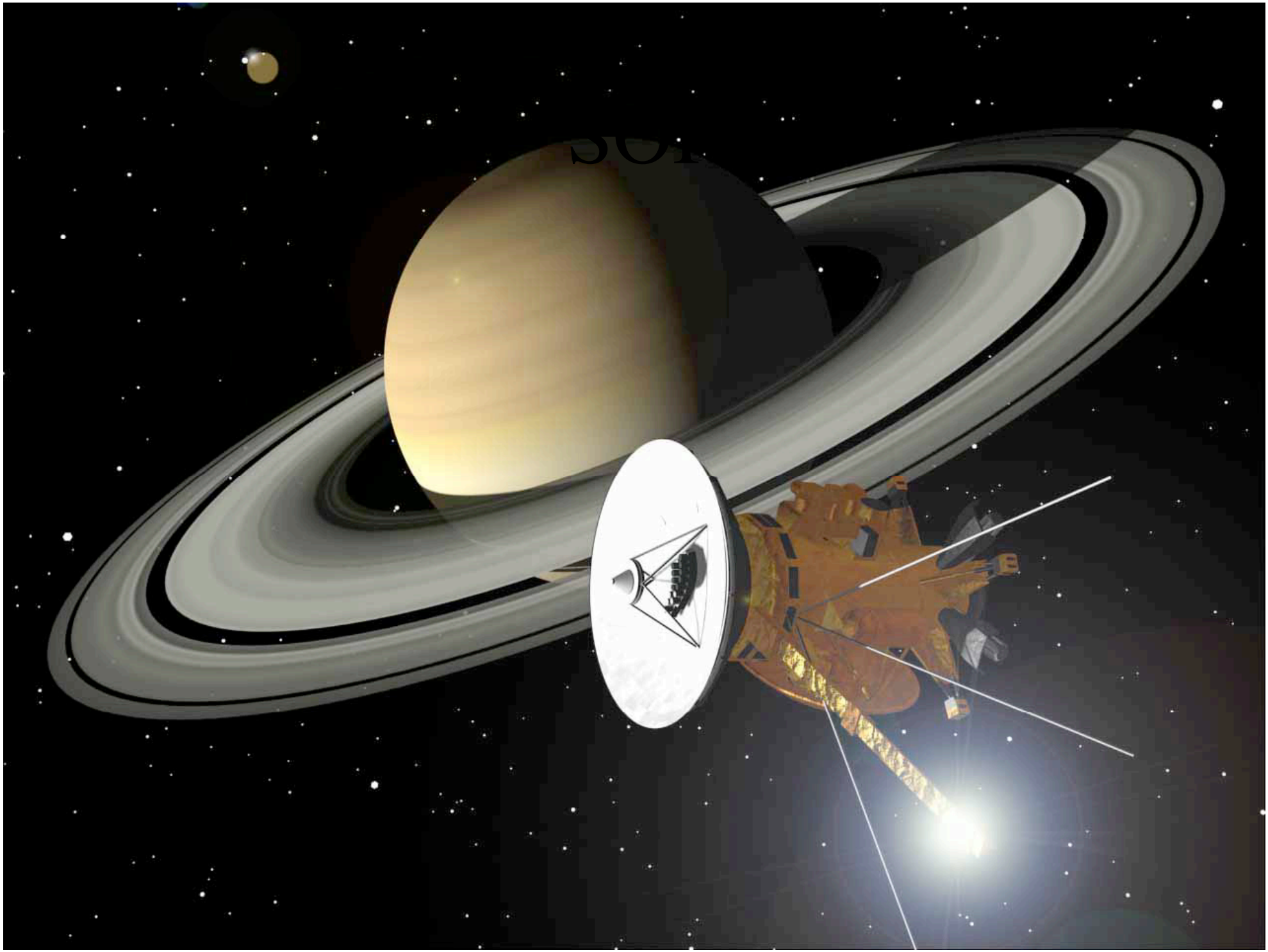


Cassini Spacecraft - 1



Cassini Spacecraft - 2





Optimization

One of the main factors limiting mission duration is the amount of propellant required to control the trajectory. It is of interest, therefore, to find maneuver strategies that accomplish the objectives with a minimum amount ofpropellant.

Rewrite the control equation as:

$$\Delta x(t_f) = \sum_{i=1}^{n} \Delta V_i K_i(t_f, t_i)\hat{\beta}_i \quad \text{where } K_i = \Phi_{x,v}^{-1}(t_f, t_i), \quad \left|\hat{\beta}_i\right| = 1, \quad \Delta V_i \text{ is the maneuver magnitude}$$

Minimize the cost function

$$J = \sum_{i=1}^{n} \Delta V_i + \overline{\lambda}^T \left[\sum_{i=1}^{n} \Delta V_i K_i \hat{\beta}_i - \Delta x \right] \qquad \text{where } \lambda \text{ is the (3x1) Lagrange multiplier}$$

Rewrite as

$$\mathrm{J} = -\lambda^{\mathrm{T}} \Delta x + \sum_{i=1}^{n} \Delta V_i \left(1 + \lambda^T K_i \hat{\beta}_i\right)$$

In order to minimize J, the thrust direction $\hat{\beta}$ at each optimal impulse point should be chosen as

$$\hat{\beta}_i^* = \frac{-K_i^T \overline{\lambda}}{\left[\overline{\lambda}^T K_i K_i^T \overline{\lambda}\right]^{1/2}}$$

λ is also called the primer vector

Optimization (Continued)

With this choice of $\hat{\beta}_i^*$ the cost function becomes:

$$J = -\lambda^T \Delta x + \sum_{i=1}^{n} \Delta V_i \left[1 - \left(\lambda^T K_i K_i^T \lambda \right)^{1/2} \right]$$

Since $\left(\lambda^T K_i K_i^T \lambda \right) > 0$, the minimum J is achieved by appling the impulses at locations where $\left(\lambda^T K_i K_i^T \lambda \right)$ is a maximum. With normalization of λ $\left(\lambda^T K_i K_i^T \lambda \right)$ can be chosen to be 1 at the optimal impulse points.

since $\left| \hat{\beta}_i^* \right| = 1$, then at each optimal impulse point

$$\lambda^T K_i K_i^T \lambda = 1 \qquad (1)$$

at each optimal impulse point and < 1 elsewhere

finally, the optimal solution must achieve the desired correction, thus

$$\Delta x = \sum_{i=1}^{n} \Delta V_i K_i K_i^T \lambda \qquad (2)$$

Linearization

Since the system of equations is non - linear, linearize about a nominal solution

Expanding equations (1) and (2) yields

$$\lambda^T K_i K_i^T \delta\lambda = \frac{1}{2}\left(1 - \lambda^T K_i K_i^T \lambda\right) \text{ for i } = 1,...n$$

$$\sum_{i=1}^{n}\left[K_i K_i^T \lambda\left(\delta\Delta V_i\right)\right] + \left[\sum_{i=1}^{n}\Delta V_i K_i K_i^T \lambda\right]\delta\lambda = \Delta x - \sum_{i=1}^{n}\Delta V_i K_i K_i^T \lambda$$

Jones, J.B.

Optimal Multi-Impulse Linear Correction of Terminal Conditions of Flyby Missions

AIAA/AAS Astrodynamics Conference

August 1978, Paper 78-1392