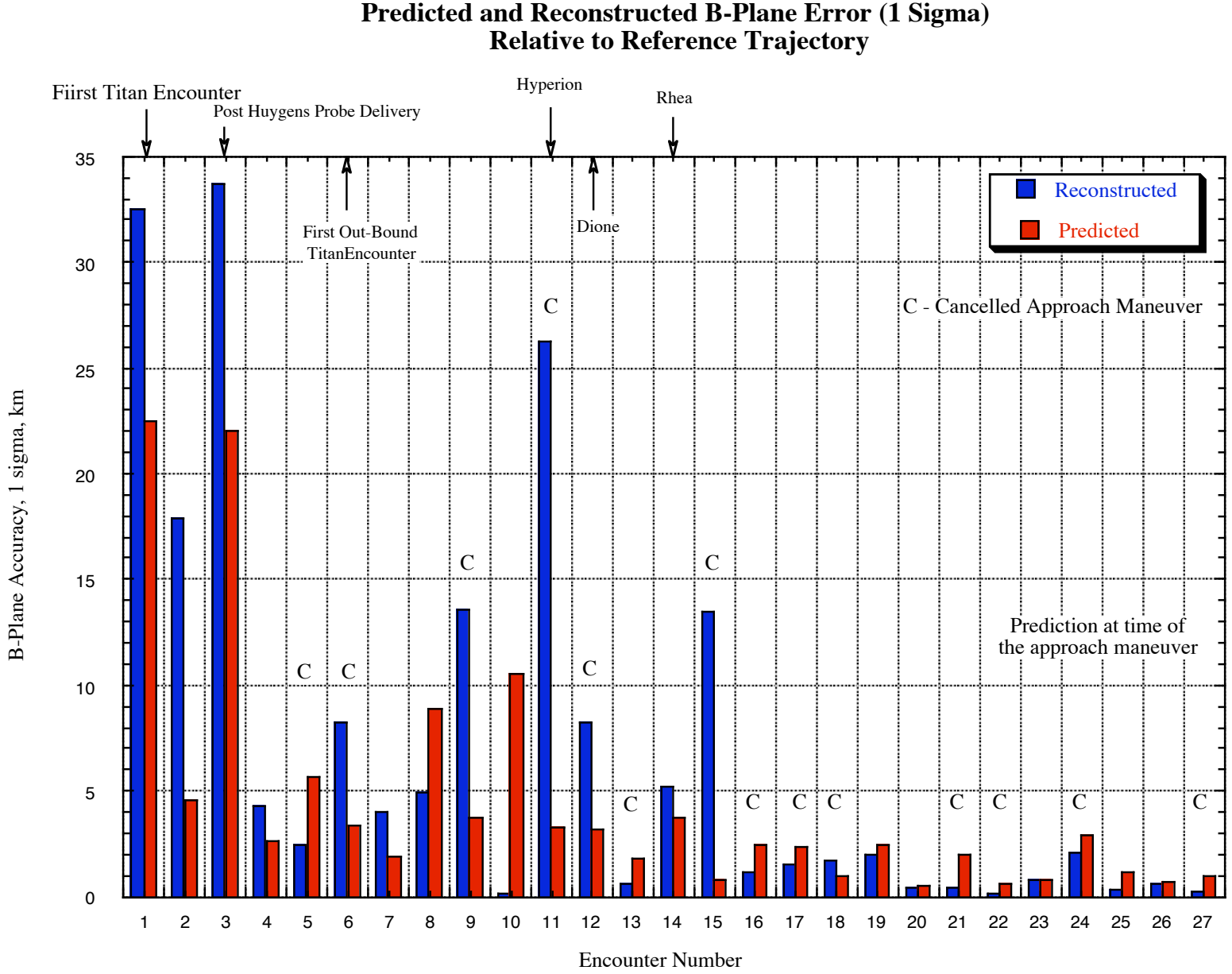
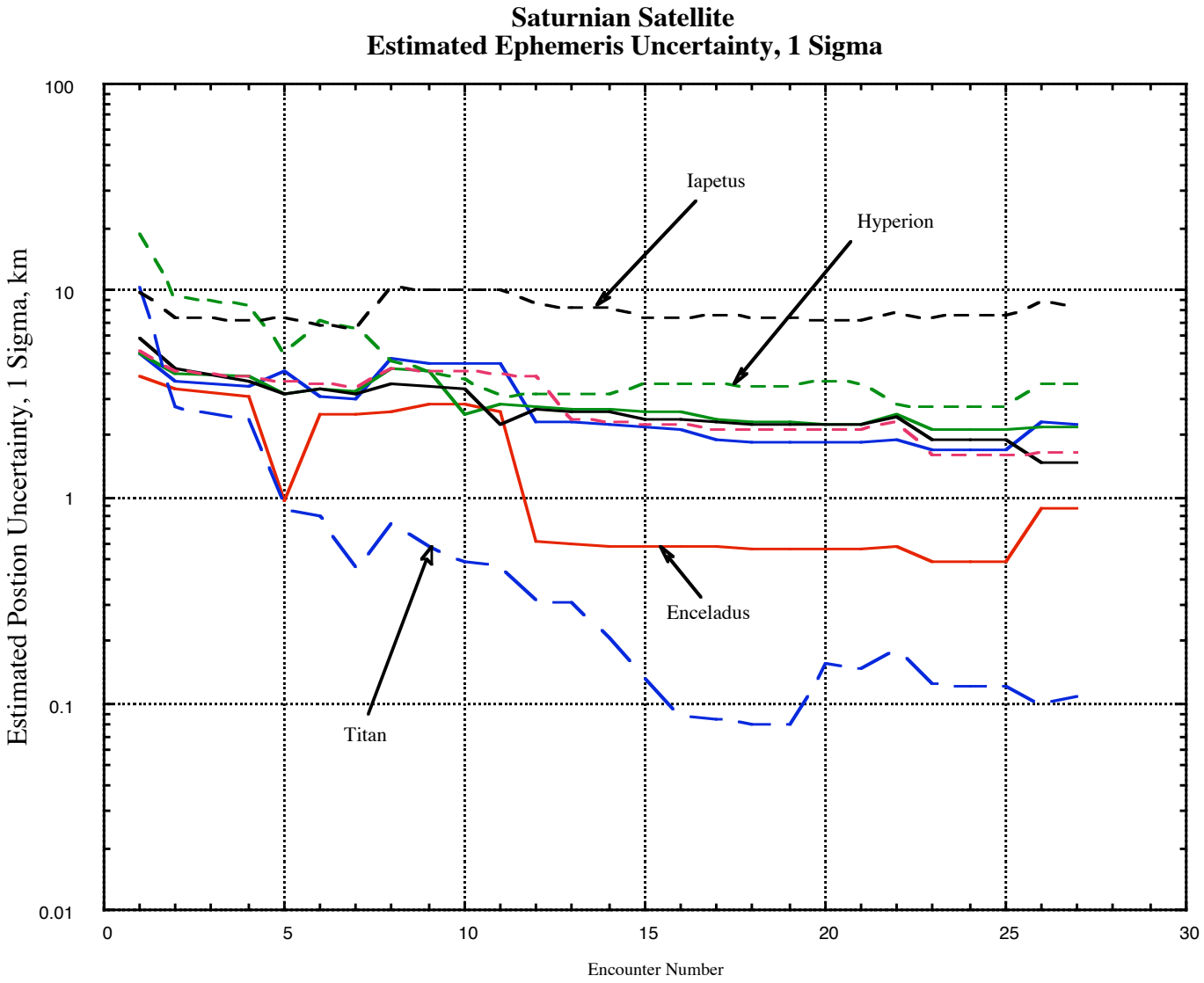
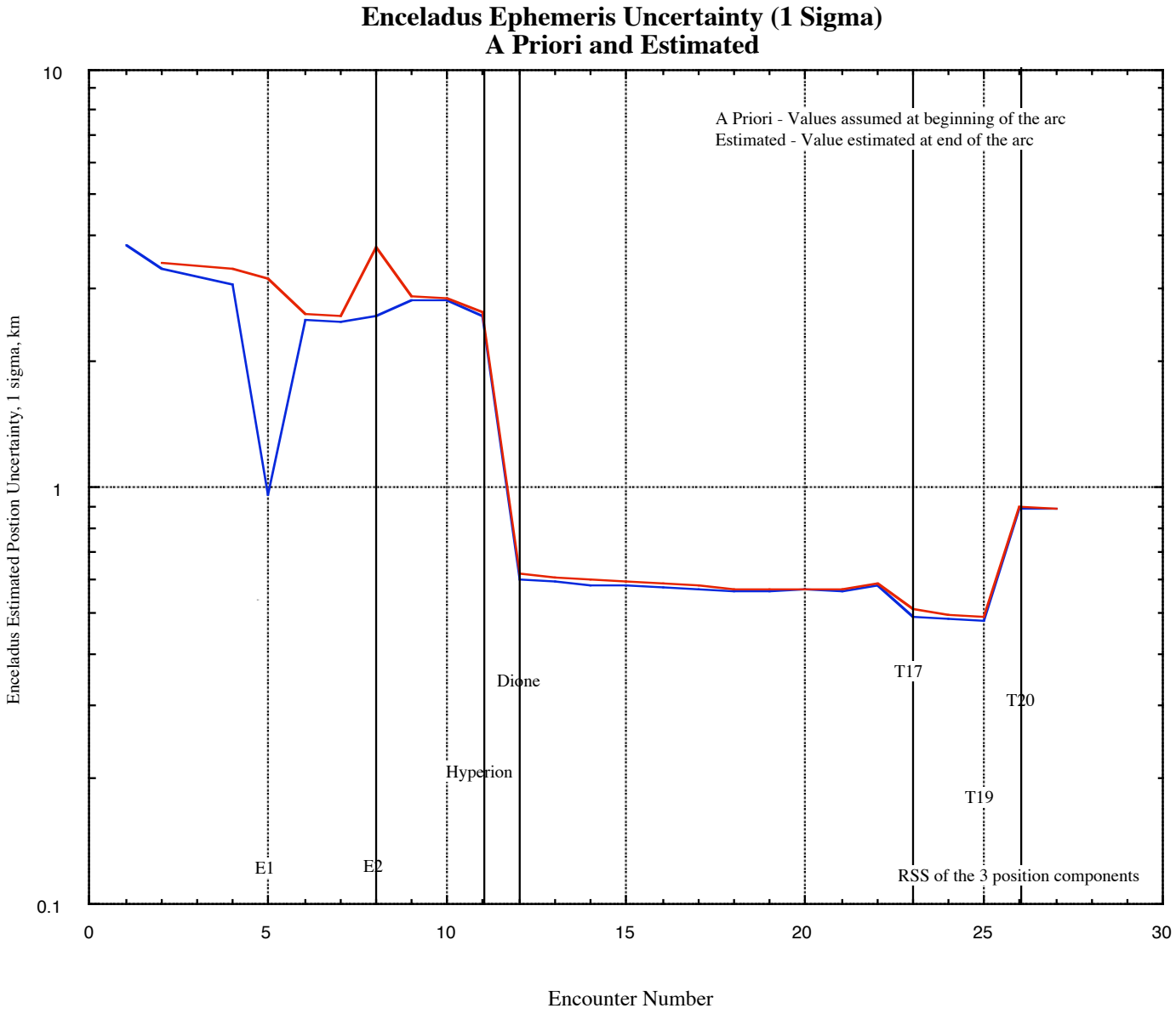

Predicted and Reconstructed B-Plane Error (1 Sigma)
Relative to Reference Trajectory
Fiirst Titan Encounter
Post Huygens Probe Delivery
First Out-Bound
TitanEncounter
Hyperion
Dione
Rhea
Reconstructed
Predicted
C - Cancelled Approach Maneuver
Prediction at time of
the approach maneuver
B-Plane Accuracy, 1 sigma, km
Encounter Number
0
5
10
15
20
25
30
35
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27
C

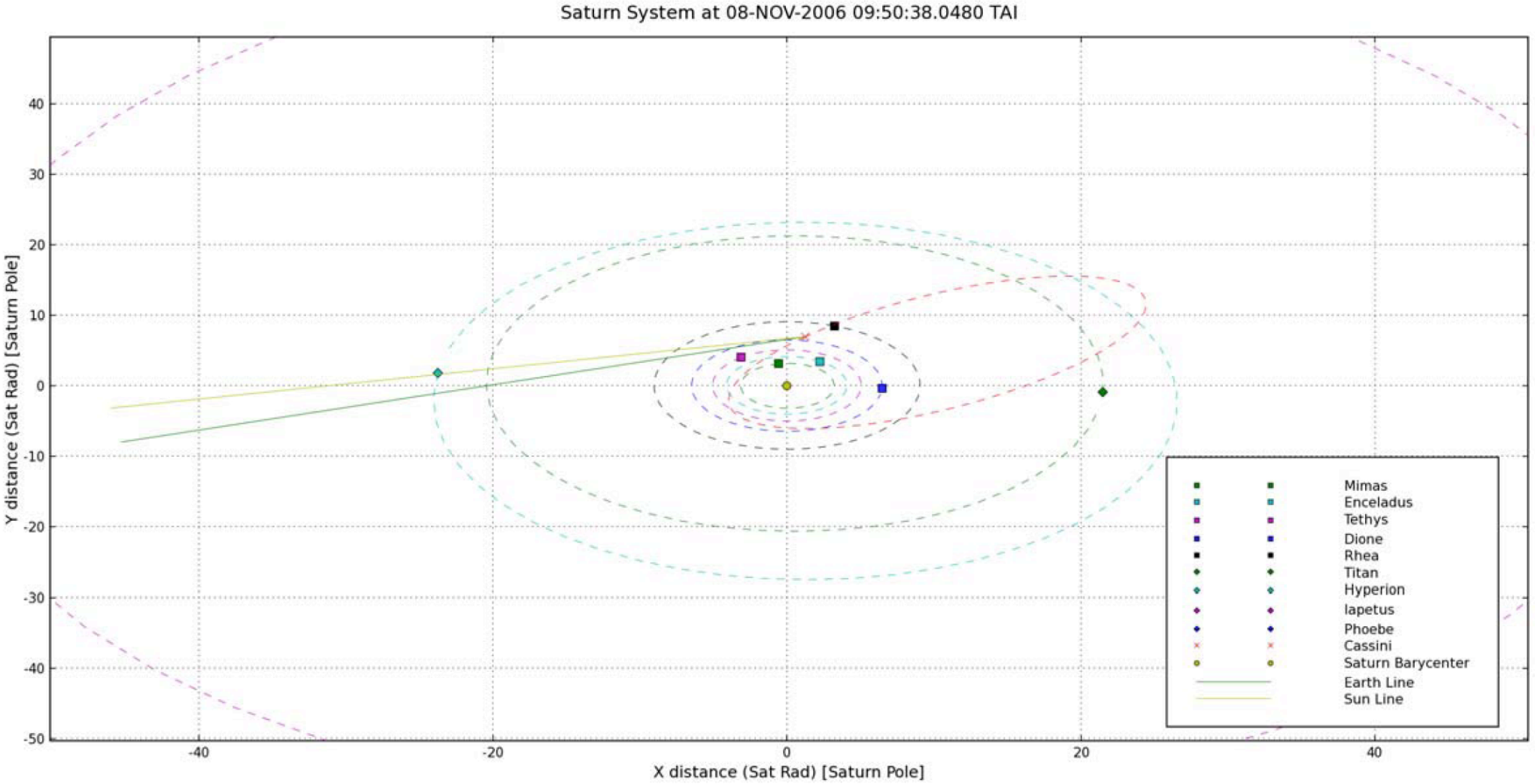
# Satellite Ephemeris Accuracy

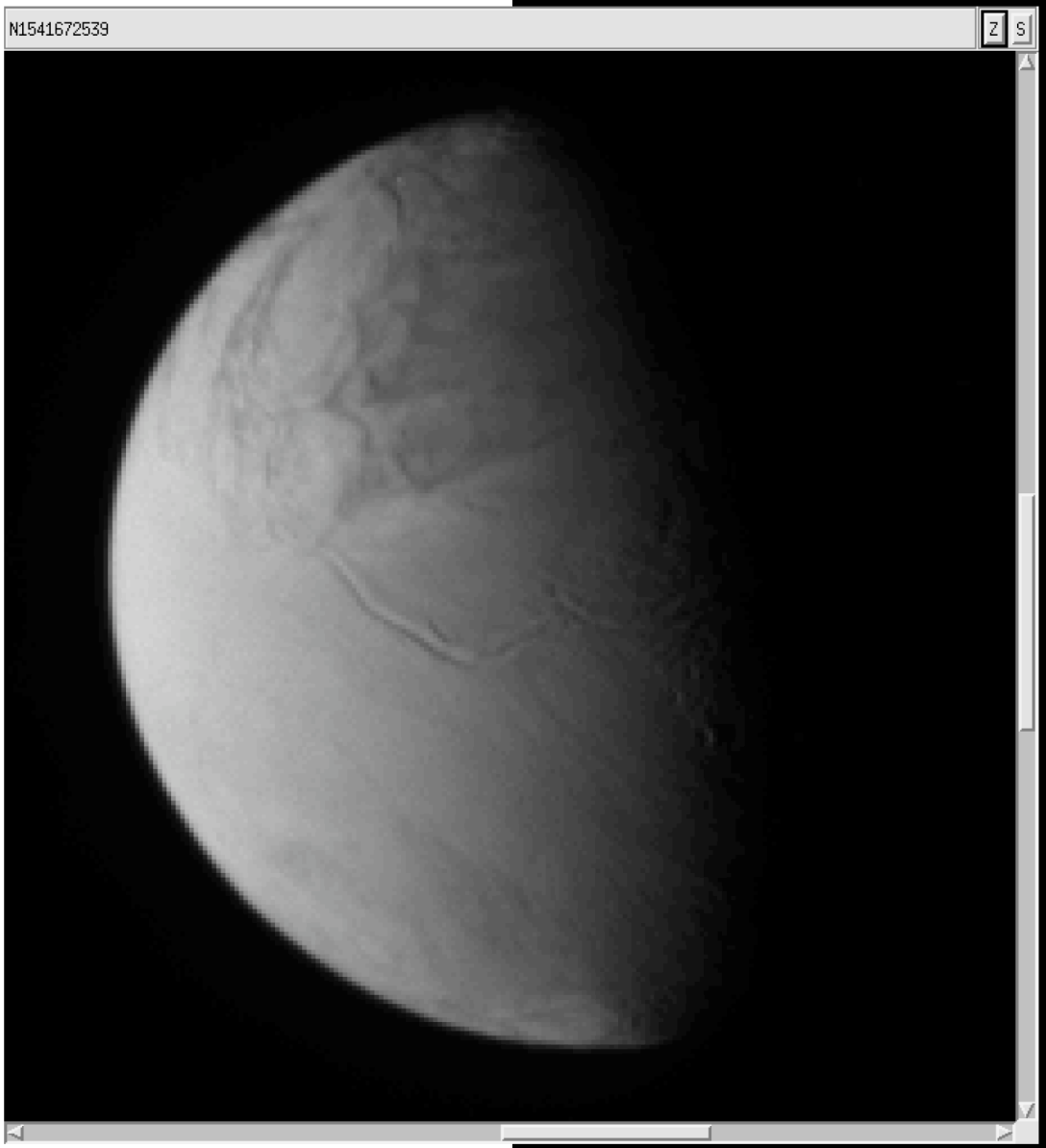


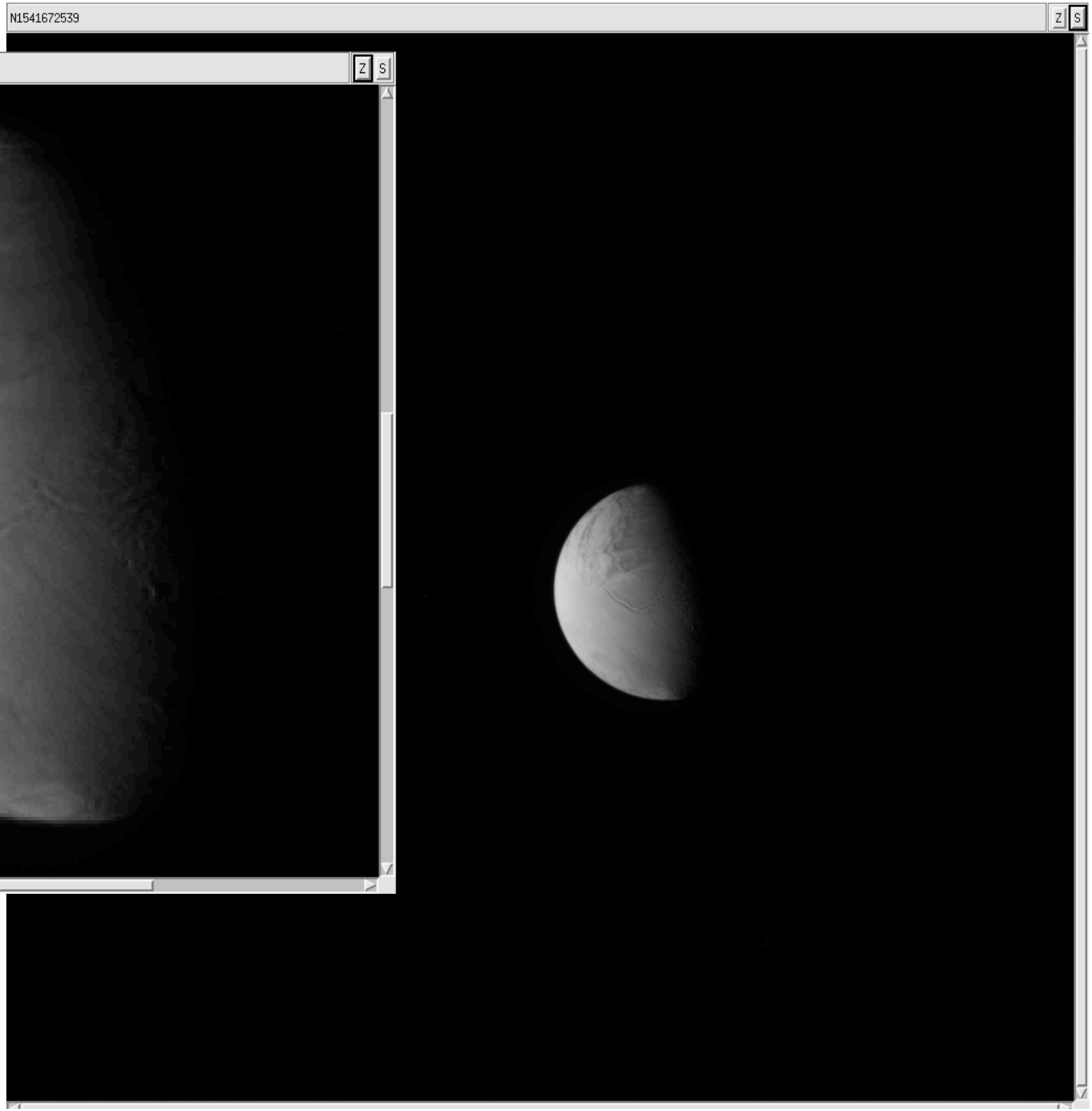
# Satellite Ephemeris Accuracy



# Optical Navigation Geometry

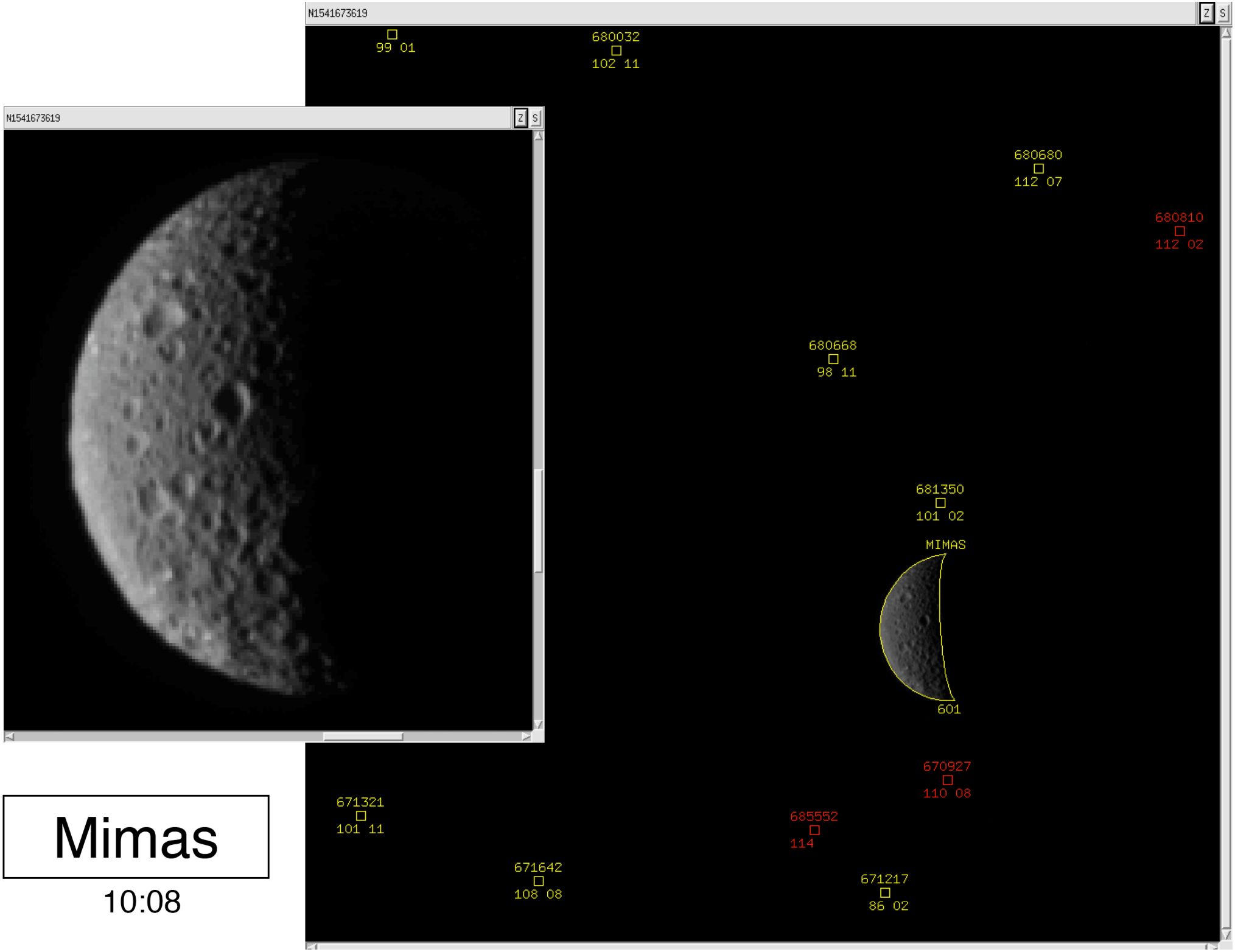





Enceladus

09:50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601
670927
110 08
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101 11
685552
114
671642
108 08
671217
86 02
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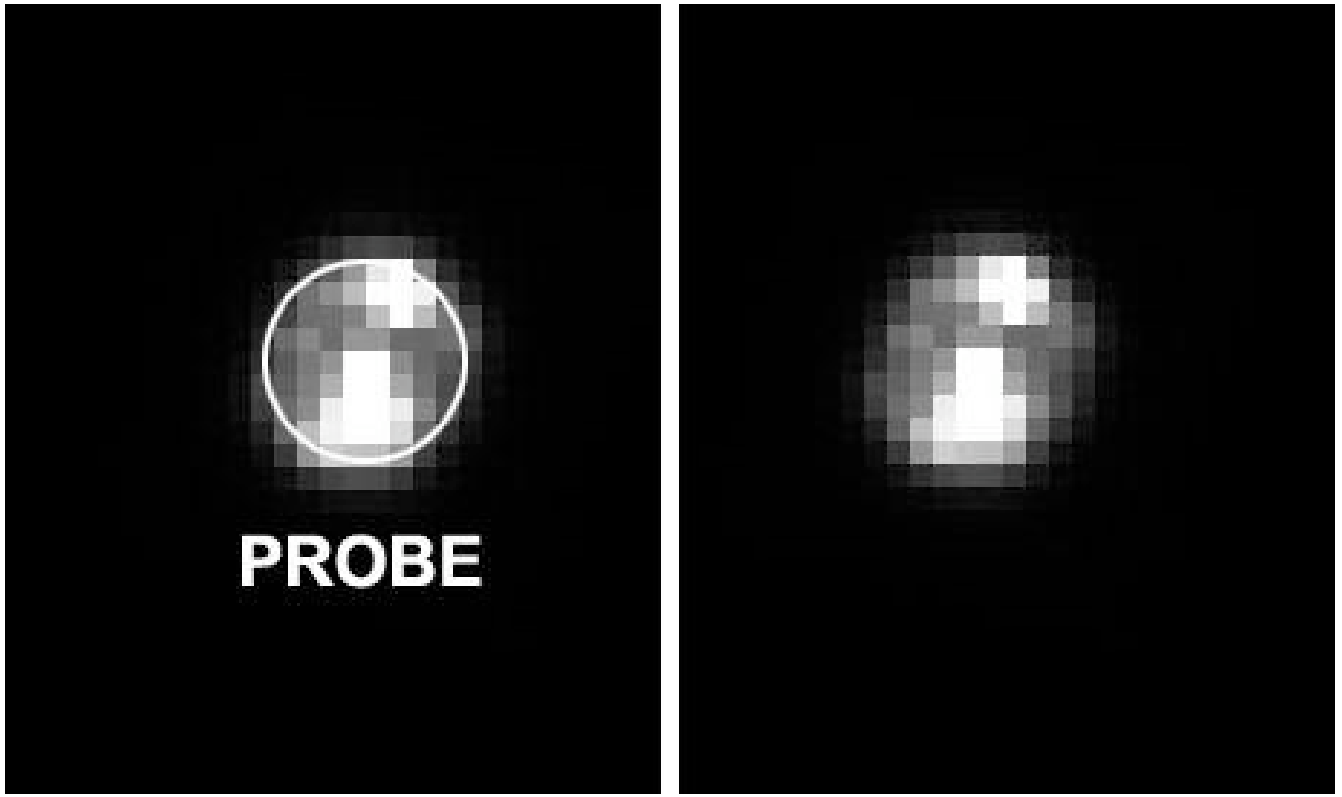
# Mimas

10:08

# Probe Images



# Control

- Typically maneuvers (changes in the spacecraft velocity) are required in order to achieve the objectives of the mission
    - Deterministic maneuvers
        - When the "free fall" trajectory does not go the desired terminal condition, a deterministic maneuver will be required,
    - Statistical maneuvers
        - Correcting errors in the trajectory resulting from:
            - Launch vehicles injection errors
            - Errors in the estimate of the orbit
            - Errors in the execution of a previous maneuver
            - Errors in a flyby

# Mathematics

Using state transition matrix the state (position and velocity) at the time $t_i$ is related to the initial state, $t_0$ by :

$$s(t_i) = \begin{bmatrix} x(t_f) \\ v(t_f) \end{bmatrix} = \Phi(t_i, t_0) s(t_0) = \Phi(t_i, t_0) \begin{bmatrix} x(t_0) \\ v(t_0) \end{bmatrix}$$

The influence of manuvers on the final state is

$$s(t_f) = \Phi(t_f, t_0) s(t_0) + \sum_i \Phi(t_f, t_i) \begin{bmatrix} 0 \\ \Delta V_i \end{bmatrix}$$

Therefore the change in the final state is;

$$\Delta s(t_f) = s(t_f) - \Phi(t_f, t_0) s(t_0) = \sum_i \Phi(t_f, t_i) \begin{bmatrix} 0 \\ \Delta V_i \end{bmatrix} = \sum_i \begin{pmatrix} \Phi_{x,x} & \Phi_{x,v} \\ \Phi_{v,x} & \Phi_{v,v} \end{pmatrix} \begin{bmatrix} 0 \\ \Delta V_i \end{bmatrix}$$

*A* single maneuver (with 3 parameters) can only control 3 parameters at the final time

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*A* single maneuver (with 3 parameters) can only control 3 parameters at the final time

# Mathematics (Continued)

If the objective is to control the position at the final time, then the required maneuver is:

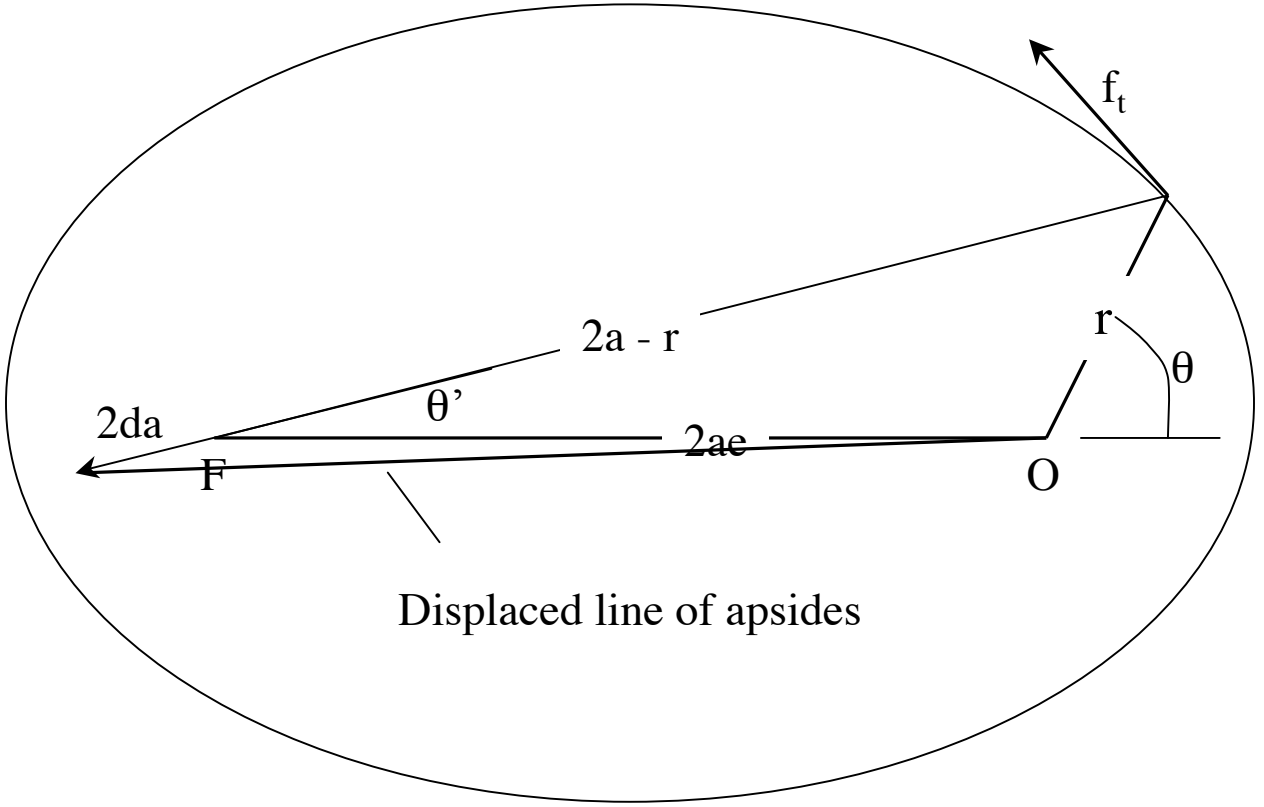
$$\Delta V = \Phi_{x,v}^{-1}(t_f, t_i)\Delta x(t_f)$$

This is the simplest maneuver strategy used in interplanetary navigation. The magnitude of the maneuver is a function of both the desired correction and the time of the maneuver.

If the requirement is control more than 3 components of the final state then multiple maneuvers will be required.

For example, if the requirement is to rendezvous with the target (match position and velocity) then a minimum of 2 maneuvers will be required.

# Orbit Control - Variation of Parameters - Tangential Impulse



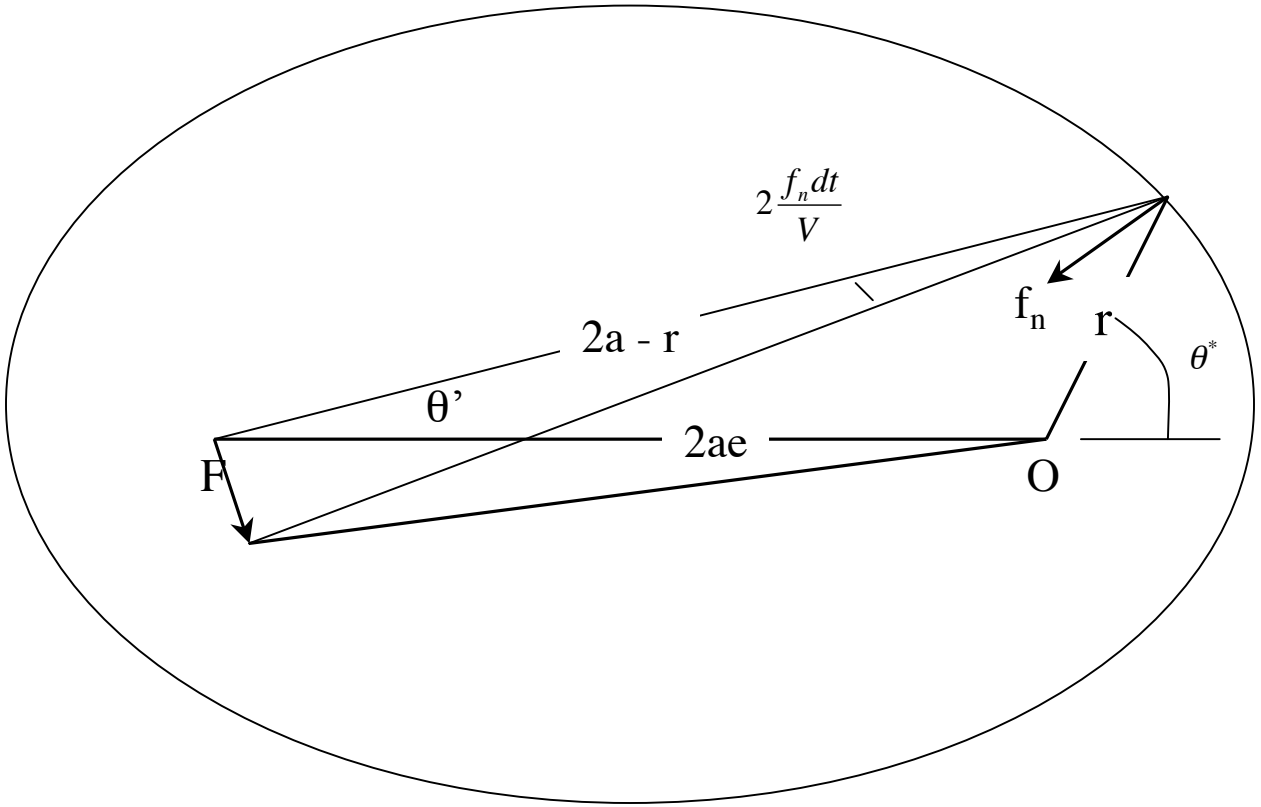

Tangential Impulse, $f_t$

$$Vf_t dt = \frac{\mu}{2a^2} da$$

Vacant focus, F, is displaced from the instaneous position by 2da

Orbit changes both energy and orientation

# Orbit Control - Variation of Parameters - Radial Impulse




Radial Impulse - $f_n dt$

*L*eaves semi - major axis unchanged

Changes direction of the tangent by $\frac{f_n dt}{V}$

The direction of the vacant focus changes by $2\frac{f_n dt}{V}$

# Orbit Control - Variation of Parameters In-Plane Result

Using geometry and orbit relationships results in

$$\dot{e} = \frac{r}{h}\left\{\sin\left(\theta^*\right)\left(1+e\cos\left(\theta^*\right)\right)f_n + \left(e+2\cos\left(\theta^*\right)+e\cos^2\left(\theta^*\right)\right)f_t\right\}$$

$$\dot{\omega} = \frac{r}{he}\left\{-\cos\left(\theta^*\right)\left(1+e\cos\left(\theta^*\right)\right)f_n + \sin\left(\theta^*\right)\left(2+e\cos\left(\theta^*\right)\right)f_t\right\}$$

where

h - angular momentum

r - radius

e - eccentricity

$\omega$ - argument of periapsis

$\theta^*$ - true anomaly

# Propulsion Systems

- Bi-Propellant
    - High thrust and efficiency
    - Cassini
        - 440 Newton thrust, Isp about 305 sec
        - Nitrogen tetroxide ($N_2O_4$), mono-methyl-hydrazine($N_2H_3CH_3$)
        - Used for maneuvers greater than about .3 m/s
- Mono-Propellant
    - Low thrust, used for attitude control and small maneuvers
    - Cassini
        - About 0.8 Newton thrust, Isp about 220 sec
        - Hydrazine
- Low thrust

# Maneuver Objectives

- Orbit maintenance
  - Control 1 or 2 orbit parameter (eccentricity, inclination )
  - Geo-stationary orbits
- Orbit transfer
  - Control 4 orbit parameters (eccentricity vector and inclination vector)
  - Transfer from low Earth orbit to a high Earth orbit
- Rendezvous
  - Control all 6 orbit parameter at specific time
  - Rendezvous of shuttle with Space Station
- Flybys
  - Flyby from one satellite encounter of another (3 parameters at a specific time)

# Maneuver Objectives

- The typical maneuver objective is to achieve the mission objective with
  - Maximum accuracy
  - Minimum propellant

- Accuracy depends upon
  - Knowledge of the state and the target at the time of the maneuver
  - Accuracy of the execution of the maneuver
  - Error growth characteristics

- Minimum Propellant
  - Number and location of the maneuvers
  - Orbit Determination and Maneuver accuracy

# Maneuver Locations

- Optimal locations
  - All maneuver strategies strive to minimize the propellant cost
  - Typical optimal maneuver locations
    - Energy change - to either change or correct the orbital energy

      The orbital energy is $E = \frac{1}{2}V^2 - \frac{\mu}{r}$

      for an impulsive maneuver, ΔV, the energy change is

      $\Delta E = V * \Delta V + \Delta V^2$

      therefore the optimal location for changing the energy is where the orbit velocity is a maximum.

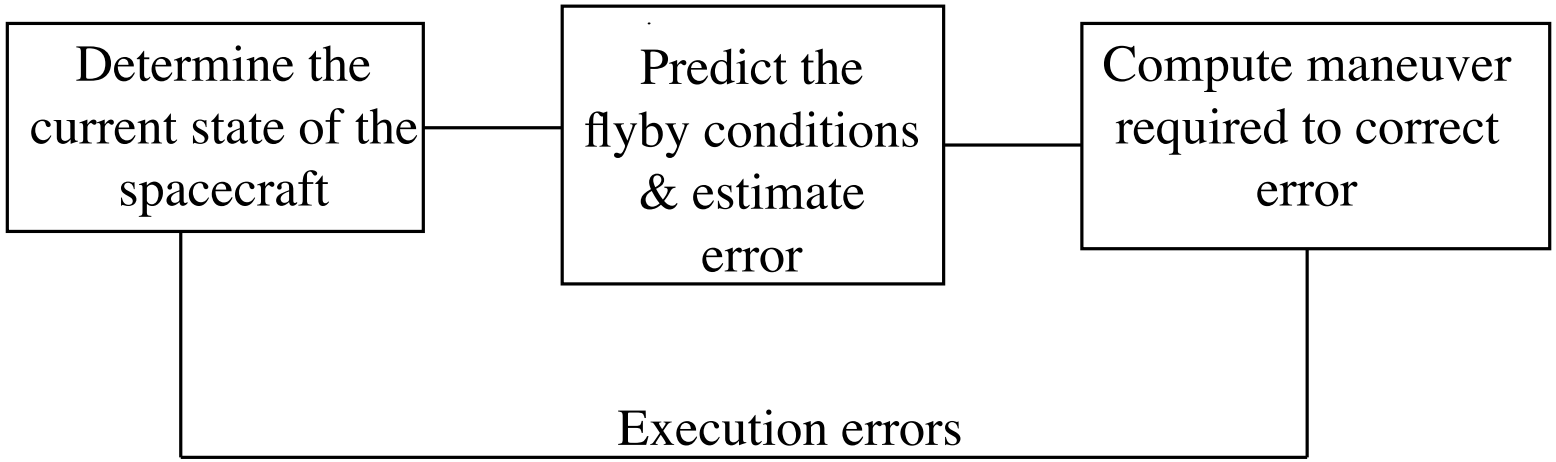
    - Inclination angle

      $\Delta i = \tan^{-1}\left(\frac{\Delta V}{V}\right)$

      therefore the maximum change in the relative inclination angle is achieved when the orbital velocity is a minimum

# Orbit Determination & Maneuver Interaction

Successful execution of mission like Cassini with many maneuvers requires continued interaction between the orbit determination process and the maneuver process.



# Statistical Navigation Analysis

- Prior to the execution of a mission a statistical analysis is conducted in order to obtain information on:
  - Propellant requirements
  - Delivery accuracy
  - Maneuver frequency
- Requires:
  - Simulation of the orbit determination accuracies along the nominal trajectory
  - Selection of the maneuver locations and targeting strategy
  - Generation of maneuver execution error models
  - Mission simulation