Cassini Navigation Inputs to Mission Design

- Spacecraft
  - Transponder requirements
    - Frequency (X-Band, Ka-Band,VLBI tones)
  - Camera requirements
    - Reliability
  - Attitude control
    - Coupled versus Uncoupled attitude control thrusters (lost this battle)
  - Propulsion system
    - Number of repressurizations (long story)
    - Maneuver durations (min/max)
  - Data storage
    - Number of optical navigation Images
- Ground System
  - Number of tracking passes (function of mission phase)
  - Ranging accuracy requirement
  - Optical Navigation image processing

Cassini Navigation Inputs to Mission Design (Continued)

- Navigation Software
    - Launch Polynomials
    - Small forces data interface
    - Automated maneuver processing
    - Solar pressure model
    - Turn ΔV prediction
    - Chained maneuver computation
    - Satellite ephemeris numerical accuracy

Estimation

- For Interplanetary Navigation this is usually referred to as Orbit Determination

Estimation

- The foundation of navigation is having a mathematical models of the physics controlling the trajectory of the spacecraft and the measurements.
- Estimation is the process of determining the “best estimates” of the parameters of the models.
    - Examples of the parameters include
        - State (position and velocity) of the spacecraft at a specified time.
        - State and mass of the planets and satellites
        - Magnitude and direction of a non-gravitational force acting on the spacecraft
        - Accuracy of a measurement type

Mathematics

- For spacecraft orbit estimation a linear least square approach is used.
- Many different formulations but they all have certain elements in common
  - Linearization - problems are non-linear and require linearization to determine a closed form solution
    - Iteration required to reach “final” solution
    - Minimization of penalty function, usually quadratic
    - Use of Gaussian statistics

Question: Why are Gaussian statistics used ?

Simple Formulation

- Estimate the n - component state vector, x

- Using p - component measurements, z

  Where the measurements have random errors, υ (independent of x)

- The measurements and the state are related by

$$z = Hx + \upsilon$$

Where H is a known (pxn) transition matrix

- Assume that the measurement errors, υ, have a zero mean, $E\{\upsilon\} = 0$,

  and have variance $E\left\{\upsilon\upsilon^{Y}\right\} = R$ (a known (pxp) positive matrix)

- The initial estimate of the state before the measurements, $\bar{x}$

  with variance $E\left\{(x - \bar{x})(x - \bar{x})^{T}\right\} = M$ (a known (nxn) positive matrix)

Applied Optimal Control

Bryson, A. E. Jr., Ho Y-C

Blaisdell Publishing Company

Simple Formulation (Continued)

Problem: Find the value, $\hat{x}$, that minimizes

$$J = \frac{1}{2}\left[\left(x-\bar{x}\right)^T M^{-1}(x-\bar{x}) + \left(z-Hx\right)^T R^{-1}\left(z-Hx\right)\right]$$

the differential is:

$$dJ = dx^T [M^{-1}(x-\bar{x}) - H^T R^{-1}(z-Hx)]$$

for $dJ = 0$, for arbitary dx^T, requires that the coefficient of dx^T must vanish,

$$\hat{x} = \bar{x} + PH^T R^{-1}(z - H\bar{x})$$

where

$$P^{-1} = M^{-1} + H^T R^{-1} H \quad \text{(nxn inverse)}$$

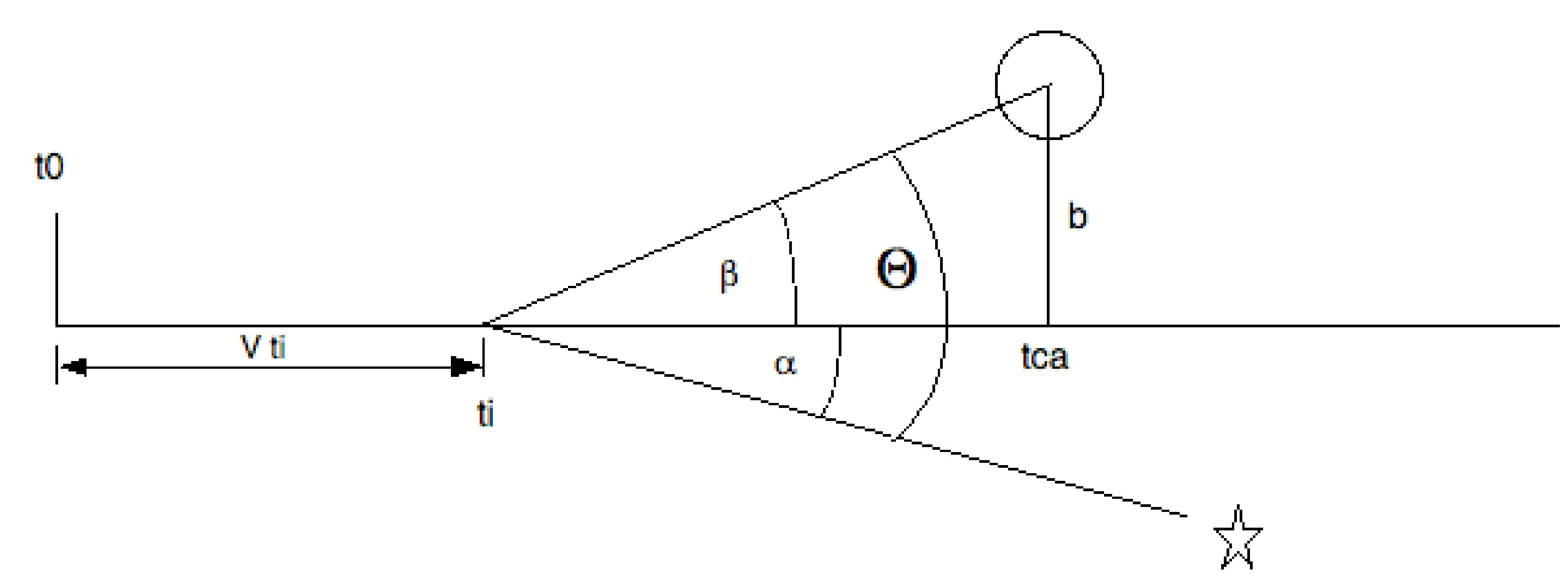
Note: for convience the follows relationships are convient if p < n:

$$P = M - MH^T\left(HMH^T + R\right)^{-1} HM \quad \text{(pxp inverse)}$$

$$PH^T R^{-1} = MH^T\left(HMH^T + R\right)^{-1} \quad \text{(pxp inverse)}$$

Simple Example

Estimate the flyby conditions for a asteroid flyby using optical navigation



Estimate the state vector $[b \;\; V \;\; t_{ca} \;\; \alpha]$

Using images at times, ti, to estimate the angle between a known star and the center of the asteroid

Formulation

The measurement, Θ at time t_i, is related to the state by:

$$\tan(\Theta) = \tan(\alpha + \beta) = \frac{b}{V(t_{ca} - t_i)}$$

therefore

$$\Theta = -\alpha + \tan^{-1}\left(\frac{b}{V(t_{ca} - t_i)}\right)$$

While the motion is linear, the measurement is non-linear, therefore linearize about an initial state estimate $\left[\bar{b}, \bar{V}, \bar{t}_{ca}, \bar{\alpha}\right]$

$$\mathrm{dz_i} = \Theta_i - \bar{\Theta}_i = \left[H_{1i}, H_{2i}, H_{3i}, H_{4i}\right] dx + \upsilon_i$$

The measurement partials at each measurement time, t_i are:

$$H(1,i) = \frac{\partial\Theta}{\partial b} = \frac{\bar{V}(\bar{t}_{ca} - t_i)}{\bar{r}^2}, \quad H(2,i) = \frac{\partial\Theta}{\partial V} = -\frac{\bar{b}(\bar{t}_{ca} - t_i)}{\bar{r}^2}, \quad H(3,i) = \frac{\partial\Theta}{\partial t_{ca}} = -\frac{\bar{b}\bar{V}}{\bar{r}^2}, \quad H(4,1) = \frac{\partial\Theta}{\partial\alpha} = 1$$

where $\bar{r}^2 = \bar{b}^2 + \bar{V}^2(\bar{t}_{ca} - t_i)^2$

Formulation (continued)

Let :

$$dz = \begin{bmatrix} \Theta_1 - \overline{\Theta}_1 \\ . \\ . \\ \Theta_{n1} - \overline{\Theta}_{n1} \end{bmatrix} \quad H = \begin{bmatrix} H_{1i}, H_{2i}, H_{3i}, H_{4i} \\ . \\ . \\ H_{1n}, H_{2n}, H_{3n}, H_{4n} \end{bmatrix} \quad dx = \begin{bmatrix} b - \overline{b} \\ V - \overline{V} \\ t_{ca} - \overline{t}_{ca} \\ \alpha - \overline{\alpha} \end{bmatrix} \quad \upsilon = \begin{bmatrix} \upsilon_{\mathrm{i}} \\ . \\ . \\ \upsilon_n \end{bmatrix}$$

Then we have

$$dz = Hdx + v$$

the measurement uncertainty is:

$$R = \begin{bmatrix} r_1, \ldots 0 \\ . \\ . \\ 0, \ldots r_n \end{bmatrix}$$

The uncertainty in the initial state is:

$$M = \begin{bmatrix} \sigma(dx_1)^2, \sigma(dx_1 dx_2), .,. \\ \sigma(dx_1 dx_2), \sigma(dx_2)^2, .,.; \\ . \\ .,.,\sigma(dx_3 dx_4), \sigma(dx_4)^2 \end{bmatrix}$$

Formulation (continued)

The best estimate of the initial state estimate is then given by:

$$d\hat{x} = d\bar{x} + PH^T R^{-1}\left(dz - Hd\bar{x}\right)$$

where $P = \left(M^{-1} + H^T R^{-1} H\right)^{-1}$

The best estimate can be obtained in two ways

1. Batch processing - a set of measurements are processes in a single batch
2. Sequential Processing - Each measurement is processed individually.

Since the mesurement process is non-linear, the initial state estimate should be updated with the "optimal" corrections until the corrections reach some minimal value

Measurements

- For interplanetary missions, the following measurements are used

  - Doppler - frequency shift in the carrier - measures the range rate from the ground antenna to the spacecraft antenna
    - Cassini uses X-Band with a typical accuracy of 0.1 mm/sec (1σ)
    - Since the Earth's motion affects the range rate, information on the right ascension and declination can be obtained.
    - Accuracy depends up the stability of the ground reference signal (clock), spacecraft down link is coherently locked to the uplink signal.
    - Major error source is the interplanetary plasma

  - Range - Measurement of the round trip light time
    - Range typical accuracy of 1 meters (1σ)
    - Path length dependent

Measurements (Continued)

- Optical Images - measurement of the angle between a known star and the optical center of a target body
  - Star location estimated to about 0.25 pixels (1σ)
  - Accuracy of the estimation of dynamical center depends very much upon the body and the phase angle
    - Typical problems are:
      - Atmosphere - Titan
      - Irregular shape - Hyperion
- VLBA - Very Long Baseline Array
  - Measures the angle spacecraft and celestial radio sources
  - Accurate estimate of the angular location of the spacecraft in the radio frame. Typical accuracy of 0.1 nanoseconds in the delay time at two stations.
  - Primarily used of Saturn ephemeris improvement
- Angles - The antenna angles
  - Typically useful when very close to the station (Earth orbiters and post-launch)

Models

- Estimating the orbit of the spacecraft requires models describing
  - The forces acting on the spacecraft
  - The measurement

Forces acting on the spacecraft

<u>Gravitation Forces</u>

- Central body
- Third body
- Harmonics
- Relativity

<u>Non-Gravitational Forces</u>

- Thrusters
  - Orbit corrections
  - Attitude control
- Solar pressure
- Thermal radiation
- Drag

Models (Continued)

<u>Measurement Models</u>

Planet and satellite ephemerides

Range biases

Troposphere & Ionosphere

Earth orientation

Station locations

Relativity

Optical pointing

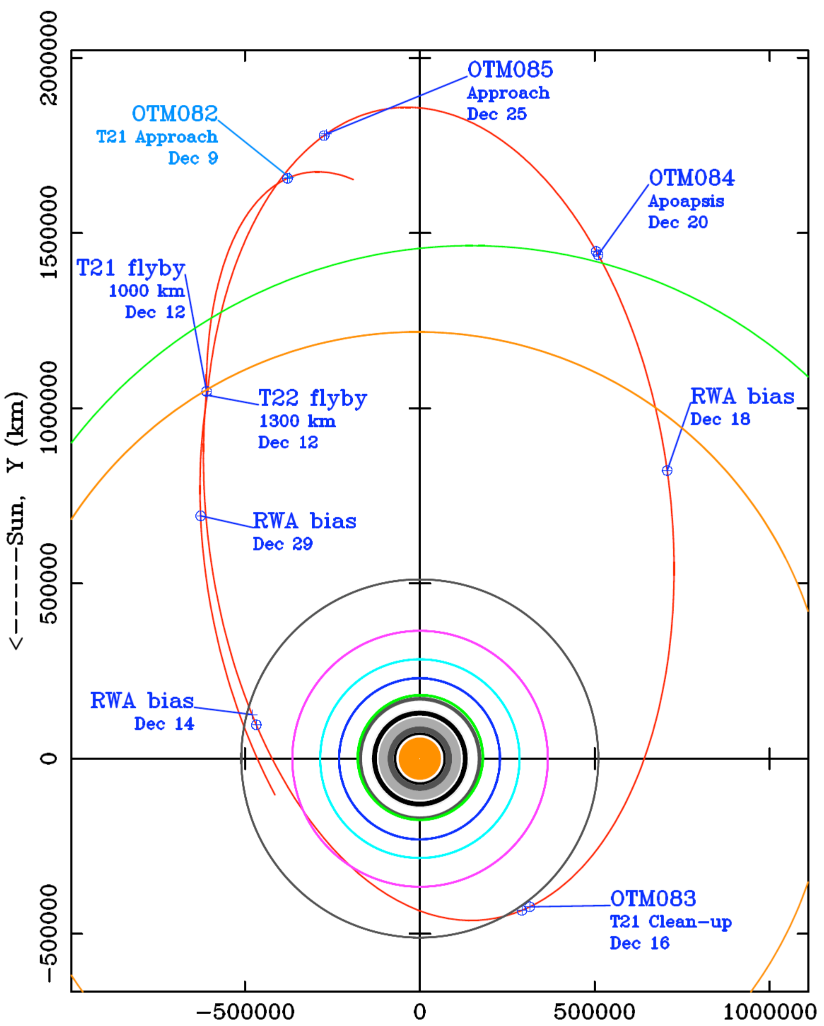
Star Catalog

Typical Cassini Orbit Determination Accuracy

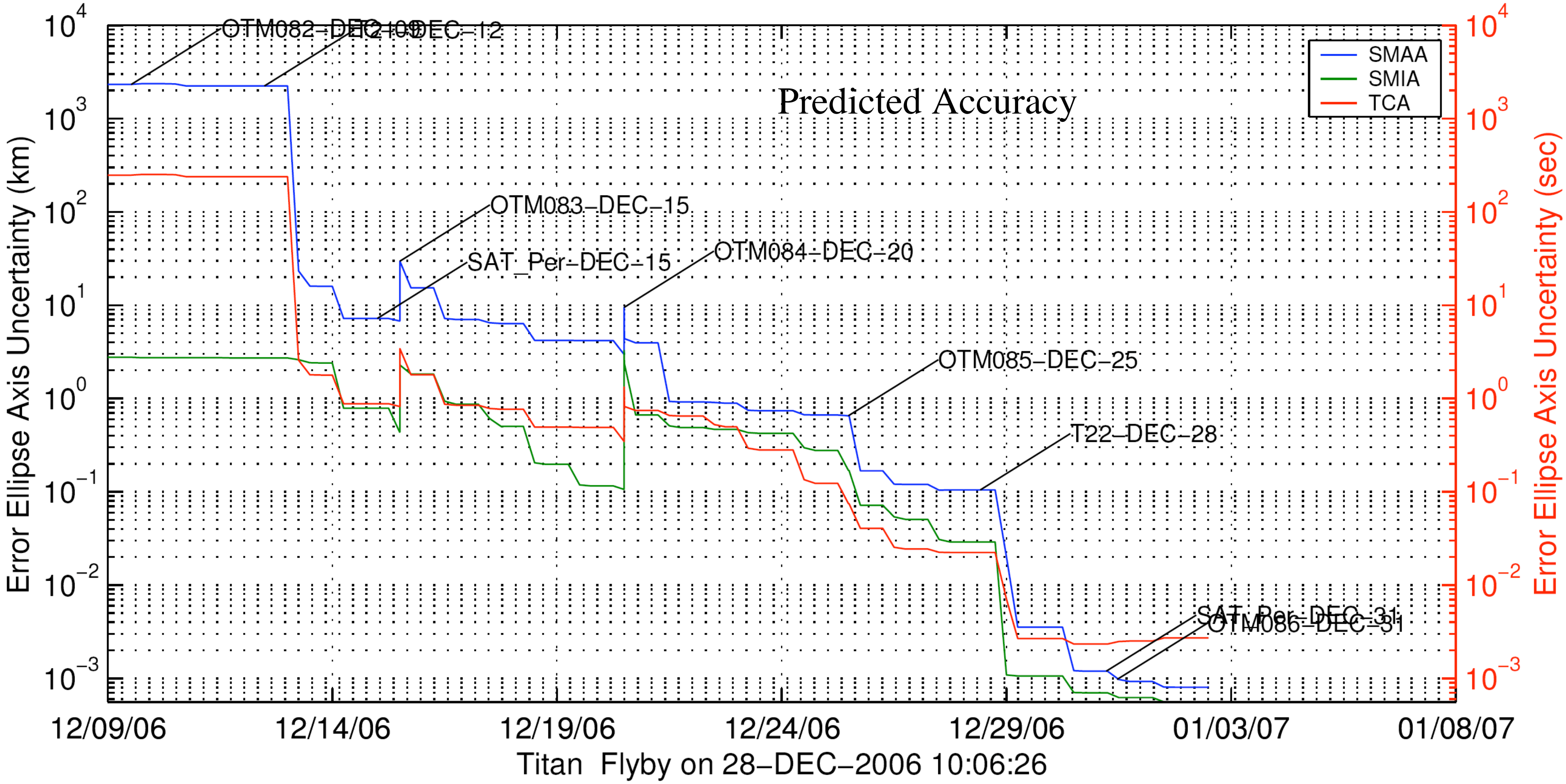
- 22th Titan Encounter (T21) to the 23st Titan encounter (T22)
- Data arc starts at the apoapsis before the T21 encounter and extends to the T22 encounter
- Three orbit trim maneuvers (OTMs) between the Titan encounters.
- 16 days between encounters
- 16 day orbit period

Titan 21 to Titan 22

16 day resonate transfer

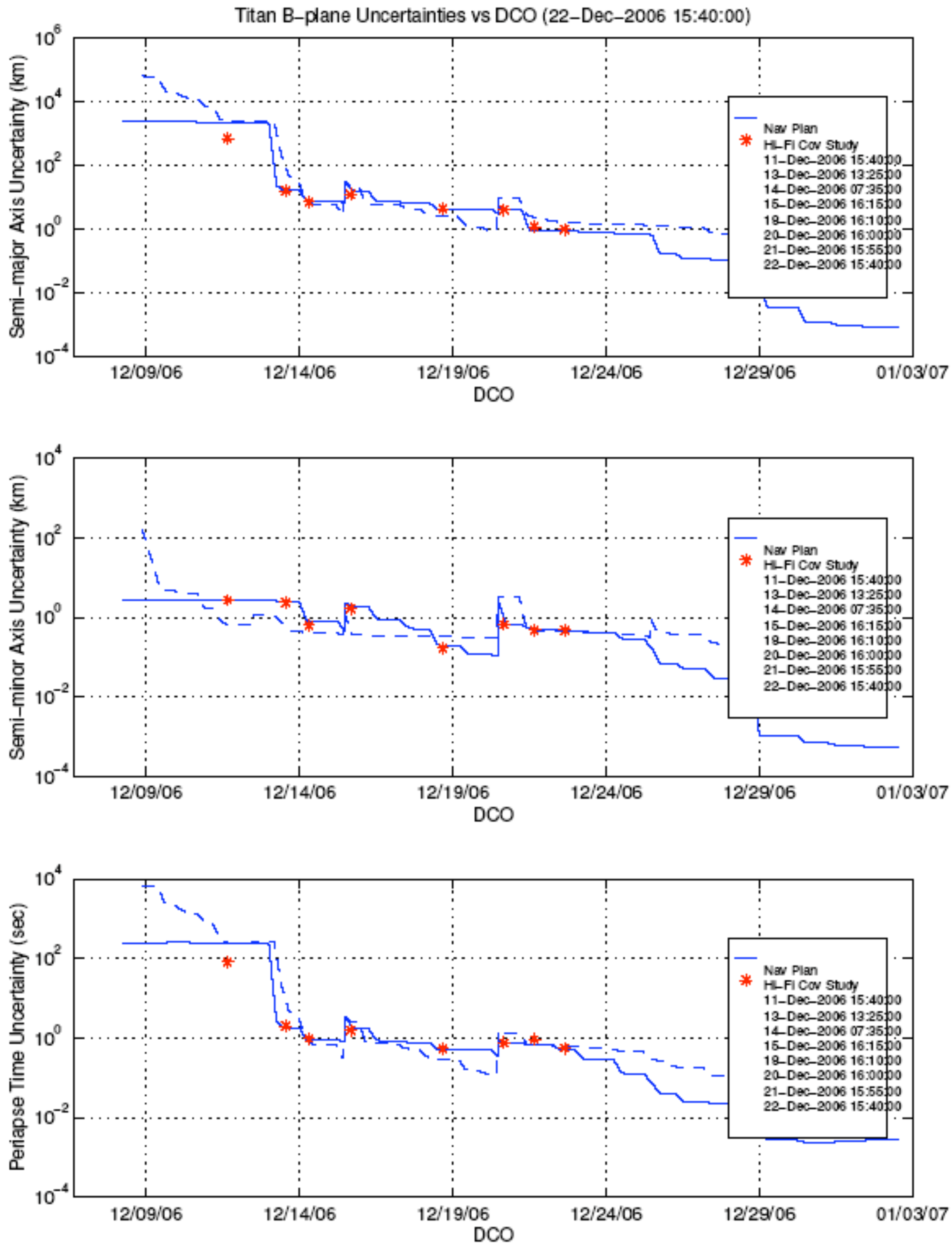


Arc Epoch (ET)	08-DEC-2006 06:00:00	
OTM082 DCO	08-DEC-2006 16:45:00	
Saturn Apo	08-DEC-2006 21:02:17	
OTM082	09-DEC-2006 12:32:00	T21 - 3 days
Titan-21	12-DEC-2006 11:41:31	1000 km, RCS
RWA bias	14-DEC-2006 02:59:10	
OTM083 DCO	14-DEC-2006 07:40:00	
Saturn Peri	15-DEC-2006 00:02:00	
OTM083	15-DEC-2006 12:03:00	T21 + 3 days
RWA bias	18-DEC-2006 10:21:10	
OTM084 DCO	19-DEC-2006 16:00:00	
OTM084	20-DEC-2006 11:48:00	Apo Mvr
Saturn Apo	22-DEC-2006 23:11:47	
OTM085 DCO	23-DEC-2006 08:15:00	
OTM085	25-DEC-2006 11:34:00	T22 - 3 days
Titan-22	28-DEC-2006 10:05:22	1300 km, RWA
RWA bias	29-DEC-2006 04:50:10	
Arc End (ET)	30-DEC-2006 12:00:00	



T22 Uncertainties at OTM DCOs (not including future maneuver uncertainties)

OTM	Name	DCO (UTC)	B·R (km)	B·T (km)	Altitude (km)	TCA (s)	SMAA (km)	SMIA (km)
OTM083	T21 CU	14-DEC 07:40:00	3.4	6.4	2.9	0.9	7.2	0.8
OTM084	Apo	19-DEC 16:00:00	2.3	3.5	2.0	0.5	4.2	0.2
OTM085	T22 APR	23-DEC 08:15:00	0.5	0.7	0.5	0.3	0.7	0.4


Titan B-plane Uncertainties vs DCO (22-Dec-2006 15:40:00)
Semi-major Axis Uncertainty (km)
Semi-minor Axis Uncertainty (km)
Periapse Time Uncertainty (sec)
DCO
12/09/06
12/14/06
12/19/06
12/24/06
12/29/06
01/03/07
Nav Plan
HI-FI Cov Study
11-Dec-2006 15:40:00
13-Dec-2006 13:25:00
14-Dec-2006 07:35:00
15-Dec-2006 16:15:00
19-Dec-2006 16:10:00
20-Dec-2006 16:00:00
21-Dec-2006 15:55:00
22-Dec-2006 15:40:00

Overall Results